International Baccalaureate
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Middle Years Programme

Mathematics guide

For use from September 2011 or January 2012

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IB mission statement

The International Baccalaureate aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect.

To this end the organization works with schools, governments and international organizations to develop challenging programmes of international education and rigorous assessment.

These programmes encourage students across the world to become active, compassionate and lifelong learners who understand that other people, with their differences, can also be right.

IB learner profile

The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.

IB learners strive to be:

Inquirers	They develop their natural curiosity. They acquire the skills necessary to conduct inquiry and research and show independence in learning. They actively enjoy learning and this love of learning will be sustained throughout their lives.
Knowledgeable	They explore concepts, ideas and issues that have local and global significance. In so doing, they acquire in-depth knowledge and develop understanding across a broad and balanced range of disciplines.
Thinkers	They exercise initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned, ethical decisions.
Communicators	They understand and express ideas and information confidently and creatively in more than one language and in a variety of modes of communication. They work effectively and willingly in collaboration with others.
Principled	They act with integrity and honesty, with a strong sense of fairness, justice and respect for the dignity of the individual, groups and communities. They take responsibility for their own actions and the consequences that accompany them.
Open-minded	They understand and appreciate their own cultures and personal histories, and are open to the perspectives, values and traditions of other individuals and communities. They are accustomed to seeking and evaluating a range of points of view, and are willing to grow from the experience.
Caring	They show empathy, compassion and respect towards the needs and feelings of others. They have a personal commitment to service, and act to make a positive difference to the lives of others and to the environment.
Risk-takers	They approach unfamiliar situations and uncertainty with courage and forethought, and have the independence of spirit to explore new roles, ideas and strategies. They are brave and articulate in defending their beliefs.
Balanced	They understand the importance of intellectual, physical and emotional balance to achieve personal well-being for themselves and others.
Reflective	They give thoughtful consideration to their own learning and experience. They are able to assess and understand their strengths and limitations in order to support their learning and personal development.

Contents

How to use this guide

This guide is for use from September 2011 or January 2012, depending on the start of the school year, and for first use in final assessment in June 2012 (northern hemisphere) and December 2012 (southern hemisphere).

This document provides the framework for teaching and learning in mathematics in the Middle Years Programme (MYP) and must be read and used in conjunction with the document *MYP: From principles into practice* (August 2008).

Introduction to MYP mathematics

> Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.
>
> David Hilbert (1862–1943)

Mathematics plays an essential role both within the school and in society. It promotes a powerful universal language, analytical reasoning and problem-solving skills that contribute to the development of logical, abstract and critical thinking. Moreover, understanding and being able to use mathematics with confidence is not only an advantage in school but also a skill for problem-solving and decision-making in everyday life. Therefore, mathematics should be accessible to and studied by all students.

Mathematics is well known as a foundation for the study of sciences, engineering and technology. However, it is also increasingly important in other areas of knowledge such as economics and other social sciences. MYP mathematics aims to equip all students with the knowledge, understanding and intellectual capabilities to address further courses in mathematics, as well as to prepare those students who will use mathematics in their workplace and life in general.

In MYP mathematics, the four main objectives support the IB learner profile, promoting the development of students who are knowledgeable, inquirers, communicators and reflective learners.

Knowledge and understanding promotes learning mathematics with understanding, allowing students to interpret results, make conjectures and use mathematical reasoning when solving problems in school and in real-life situations.

Investigating patterns supports inquiry-based learning. Through the use of investigations, teachers challenge students to experience mathematical discovery, recognize patterns and structures, describe these as relationships or general rules, and explain their reasoning using mathematical justifications and proofs.

Communication in mathematics encourages students to use the language of mathematics and its different forms of representation, to communicate their findings and reasoning effectively, both orally and in writing.

Reflection in mathematics provides an opportunity for students to reflect upon their processes and evaluate the significance of their findings in connection to real-life contexts. Reflection allows students to become aware of their strengths and the challenges they face as learners.

Overall, MYP mathematics expects all students to appreciate the beauty and usefulness of mathematics as a remarkable cultural and intellectual legacy of humankind, and as a valuable instrument for social and economic change in society.

This guide provides both MYP teachers and students with:

- the requirements of the course
- strategies to incorporate the areas of interaction into mathematics
- aims and objectives for MYP mathematics
- the prescribed curriculum framework
- details of final assessment requirements, including moderation and monitoring of assessment.

IB-produced teacher support material (TSM) is available to complement this guide and aid the implementation of the course in schools.

The IB mathematics continuum

MYP mathematics builds on experiences in mathematics learning that students have gained in their time in the IB Primary Years Programme (PYP). PYP teaching and learning experiences challenge students to be curious, ask questions, explore and interact with the environment physically, socially and intellectually to construct meaning and refine their understanding. The use of structured inquiry is a precursor to the problem-solving and inquiry-based approach of MYP mathematics. Students continuing on to the IB Diploma Programme (DP) will have developed not only an inquiring and reflective approach to mathematics learning but also critical-thinking and problem-solving skills, which they will be able to apply and extend in further DP mathematics courses. In particular, the MYP framework for mathematics reflects the concepts and skills of the presumed knowledge for the DP mathematics courses at standard level (SL) and higher level (HL). The two levels of the MYP mathematics courses (standard and extended) have been refined to allow a smooth transition from MYP mathematics to DP mathematics courses.

When planning a transition from MYP to DP mathematics courses, teachers and mathematics departments are encouraged to refer to the document *Mathematics: The MYP–DP continuum*.

Aims and objectives

Aims

The aims of any MYP subject state in a general way what the teacher may expect to teach or do, and what the student may expect to experience or learn. In addition, they suggest how the student may be changed by the learning experience.

The aims of the teaching and study of MYP mathematics are to encourage and enable students to:

- enjoy mathematics and to develop curiosity as well as an appreciation of its elegance and power
- develop an understanding of the principles and nature of mathematics
- communicate clearly and confidently in a variety of contexts
- develop logical, critical and creative thinking, and patience and persistence in problem solving
- develop power of generalization and abstraction
- apply and transfer skills to a wide range of situations including real life, other areas of knowledge and future developments
- appreciate how developments in technology and mathematics have influenced each other
- appreciate the moral, social and ethical implications arising from the work of mathematicians and the applications of mathematics
- appreciate the international dimension in mathematics through an awareness of the universality of mathematics and its multicultural and historical perspectives
- appreciate the contribution of mathematics to other areas of knowledge
- develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics
- develop the ability to reflect critically upon their own work and the work of others.

Objectives

The objectives of any MYP subject state the specific targets that are set for learning in the subject. They define what the student will be able to accomplish as a result of studying the subject.

These objectives relate directly to the assessment criteria found in the "Mathematics assessment criteria" section.

A Knowledge and understanding

Knowledge and understanding are fundamental to studying mathematics and form the base from which to explore concepts and develop problem-solving skills. Through knowledge and understanding, students develop mathematical reasoning to make deductions and solve problems.

At the end of the course, students should be able to:

- know and demonstrate understanding of the concepts from the five branches of mathematics (number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics)
- use appropriate mathematical concepts and skills to solve problems in both familiar and unfamiliar situations, including those in real-life contexts
- select and apply general rules correctly to make deductions and solve problems, including those in real-life contexts.

B Investigating patterns

Investigating patterns allows students to experience the excitement and satisfaction of mathematical discovery. Working through investigations encourages students to become risk-takers, inquirers and critical thinkers.

The ability to inquire is invaluable in the MYP and contributes to lifelong learning.

Through the use of mathematical investigations, students are given the opportunity to apply mathematical knowledge and problem-solving techniques to investigate a problem, generate and/or analyse information, find relationships and patterns, describe these mathematically as general rules, and justify or prove them.

At the end of the course, students should be able to:

- select and apply appropriate inquiry and mathematical problem-solving techniques
- recognize patterns
- describe patterns as relationships or general rules
- draw conclusions consistent with findings
- justify or prove mathematical relationships and general rules.

C Communication in mathematics

Mathematics provides a powerful and universal language. Students are expected to use mathematical language appropriately when communicating mathematical ideas, reasoning and findings—both orally and in writing.

At the end of the course, students should be able to communicate mathematical ideas, reasoning and findings by being able to:

- use appropriate mathematical language in both oral and written explanations
- use different forms of mathematical representation
- communicate a complete and coherent mathematical line of reasoning using different forms of representation when investigating problems.

Students are encouraged to choose and use information and communication technology (ICT) tools as appropriate and, where available, to enhance communication of their mathematical ideas. Some of the possible ICT tools used in mathematics include spreadsheets, graph plotter software, dynamic geometry software, computer algebra systems, mathematics content-specific software, graphic display calculators (GDC), word processing, desktop publishing, graphic organizers and screenshots.

D Reflection in mathematics

MYP mathematics encourages students to reflect upon their findings and problem-solving processes.

Students are encouraged to examine different problem-solving strategies and share their thinking with teachers and peers. Critical reflection in mathematics helps students gain insight into their strengths and weaknesses as learners and to appreciate the value of errors as powerful motivators to enhance learning and understanding.

At the end of the course, students should be able to:

- explain whether their results make sense in the context of the problem
- explain the importance of their findings in connection to real life where appropriate
- justify the degree of accuracy of their results where appropriate
- suggest improvements to the method when necessary.

Requirements

MYP mathematics is a compulsory component of the MYP in **every** year of the programme.

Organizing mathematics in the school

Schools are responsible for developing their MYP mathematics curriculum so that the final aims and objectives set by the IB can be met successfully at the end of the programme. The MYP allows schools great flexibility in the way they structure and schedule their courses so that these also meet the requirements of their local and national systems.

Teaching hours

It is essential that teachers be allowed the number of teaching hours necessary to meet the requirements of the MYP mathematics course. Although the prescribed minimum teaching time in any given year for each subject group is 50 teaching hours, the IB recognizes that, in practice, more than 50 teaching hours per year will be necessary not only to meet the programme requirements over the five years, but also to allow for the sustained, concurrent teaching of disciplines that enables interdisciplinary study.

Schools must ensure that students are given sufficient time and instruction to allow them the opportunity to meet the final aims and objectives for mathematics.

Framework for mathematics

MYP mathematics provides a framework of concepts and skills organized into the following five branches of mathematics.

- **Number**
- **Algebra**
- **Geometry and trigonometry**
- **Statistics and probability**
- **Discrete mathematics**

Schools are required to structure their mathematics curriculum so that the five branches, as described in the framework, are addressed over the five years (or complete duration) of the programme.

Schools are expected to use the framework for mathematics as a tool for curriculum mapping to assist them in the vertical and horizontal planning of their courses and in the development of units of work in mathematics. There is no prescription for a particular order or sequence in which the branches of the framework should be addressed, or the way in which the concepts and skills should be used when structuring units of work in mathematics. Schools are given the opportunity to develop their courses and structure their units of work to suit their own preferences and students' needs.

However, **over the five years of the programme**, schools **must** ensure that they provide students with the opportunity to experience learning in **all** the branches of the framework, ensuring that the aims and objectives of MYP mathematics are not compromised.

Levels of mathematics

MYP mathematics should be accessible to and be studied by all students. Schools must ensure that the mathematics curriculum allows **all** students the opportunity to reach their full potential and achieve the final aims and objectives of MYP mathematics. In order to achieve this, the concepts and skills of the framework for mathematics are organized so that students can work at two levels of ability: **standard mathematics** and **extended mathematics**.

Standard mathematics aims to give all students a sound knowledge of basic mathematical concepts while allowing them to develop the skills needed to meet the objectives of MYP mathematics.

Extended mathematics consists of the standard mathematics framework supplemented by additional concepts and skills. This level provides the foundation for students who wish to pursue further studies in mathematics, for example, mathematics higher level (HL) as part of the IB Diploma Programme.

IB validation of students' grades and certification are available for both standard and extended mathematics. Schools may decide to offer one or both levels, and will then allocate students to the appropriate level.

The assessment criteria for mathematics, directly addressing the aims and objectives of the course, apply to both levels. For examples of how to apply these criteria when assessing students' work, please refer to the mathematics teacher support material (TSM) that complements this guide.

Differentiated instruction and special educational needs

It is acknowledged that not all students learn mathematics at the same speed, in the same manner, or respond in the same way to the same teaching strategies. Students of the same year level may differ substantially in their mathematical abilities, as well as in their background and previous mathematical experiences. They may also have different interests and exhibit preferred ways of learning. However, it is important that all students are provided with a positive learning experience in mathematics and have the opportunity to maximize their potential.

In mixed-ability classrooms, teachers have to differentiate their instruction and adapt their assessment tasks to meet the wide range of skills and capabilities. It is the responsibility of schools and teachers to develop teaching and learning strategies that allow all students the opportunity to work towards meeting the final objectives of MYP mathematics.

There are a number of ways in which teachers can differentiate their instruction. Teachers may:

- examine the course content and determine what essential understanding is required for different students
- focus on the outcomes and allow for different ways to demonstrate understanding
- assess how space, time and resources can be best used to create effective conditions to enhance learning for all students.

For further information and support on differentiated instruction and how to create an environment that is inclusive of students with special educational needs (SEN), please refer to the SEN page, SEN resources and forums on the OCC or contact sen@ibo.org.

Resources

The resources and tasks used should be carefully chosen and prepared so that the objectives can be met and the assessment criteria can be applied. The choice of resources within a school will also need to reflect the ability range within that school.

Library

Schools should provide teachers and students with a good variety of resources to support teaching and learning in mathematics. A well-resourced and up-to-date library equipped with books, magazines and multimedia, and which reflects the ability range within the school, can contribute to sustaining students' curiosity and stimulating their interest.

Information and communication technology (ICT)

The appropriate use of computers, computer applications and calculators can improve the understanding of all students.

Depending upon the school resources, ICT should be used whenever appropriate:

- as a means of expanding students' knowledge of the world in which they live
- as a channel for developing concepts and skills
- as a powerful communication tool.

ICT provides a wide range of resources and applications for teachers to explore in order to enhance teaching and learning.

In mathematics, ICT can be used as a tool to perform complicated calculations, solve problems, draw graphs, and interpret and analyse data. ICT can also be helpful to:

- investigate data and mathematical concepts
- obtain rapid feedback when testing out solutions
- observe patterns and make generalizations
- move between analytical and graphical representation
- visualize geometrical transformations.

In addition, the appropriate use of ICT can enhance students' communication skills, assisting them in the collection, organization and analysis of information and in the presentation of their findings.

However, for ICT to be a useful tool for learning, students need to be familiar with the resources and applications, and know how and when to use them. Students should be able to decide when the use of ICT is appropriate and when alternative methods such as pencil and paper, mental calculation, or diagrams should be used. Therefore, it is important that teachers show students how to use these resources effectively while supporting the development of their intellectual skills.

ICT can support students with special educational needs who have difficulties understanding a particular concept or who would benefit from further practice. It can also provide the extra challenge for gifted and talented students to explore further ideas and concepts. "Adaptive technologies" can enable students with severe learning disabilities to become active learners in the classroom alongside their peers. For more information about adaptive technologies and special educational needs, please refer to the SEN page on the OCC.

Depending on the school facilities and availability of ICT resources, teachers are encouraged to use ICT whenever possible and appropriate as a means of enhancing learning.

Some of the possible ICT resources in mathematics might include:

- databases and spreadsheets
- graph plotter software
- dynamic geometry software
- computer algebra systems

- programming languages
- mathematics content-specific software
- graphic display calculators (GDC)
- internet search engines
- CD-ROMS
- word processing or desktop publishing
- graphic organizers.

Language of instruction

In schools where the language of instruction of mathematics is not the mother tongue of some of the students taking the course, measures must be implemented to ensure that these students are not disadvantaged and have the full opportunity to demonstrate the highest achievement level in the final objectives. These measures may include:

- teacher training
- modification of language in materials
- differentiation of assessment tasks
- parallel resources in students' mother tongues.

For further information, please refer to the document *Learning in a language other than mother tongue in IB programmes*.

Professional development

To support teachers in meeting the aims and objectives of MYP mathematics, professional development must be carefully planned within the school. Opportunities to attend in-school workshops and IB regional conferences should be provided, to ensure that teachers develop a good understanding of the underpinning philosophy of the MYP and of the requirements of MYP mathematics in particular.

The online curriculum centre (OCC)

The OCC is a valuable resource for teachers in the MYP. Teachers are encouraged to participate in and contribute to this resource as a means of developing the IB online learning community. The OCC contains discussion forums and resource banks for all MYP subject groups, the personal project, special educational needs and academic honesty.

IB-appointed faculty members answer queries and provide advice on teaching and learning, implementation and moderation. Teachers can post queries, share resources and download all IB official publications. Please see your MYP coordinator for a school code and password.

Developing the curriculum

Introduction

All MYP subjects, including mathematics, provide a curricular framework with set final aims and objectives. Schools are responsible for developing and structuring their mathematics courses so that they provide opportunities for students to meet the final aims and objectives effectively by the end of the programme.

Teachers are expected to map the teaching and learning experiences that students will encounter as they move from one year to the next in the programme. The MYP mathematics courses should be carefully sequenced and articulated so that they contribute to the development of students' conceptual understanding, practical and intellectual skills as well as personal beliefs and values.

The MYP requires schools to facilitate and promote collaborative planning for the purpose of curriculum planning, review and reflection.

The staff responsible for teaching and learning in mathematics will need to determine the subject content for each year of the programme to make sure the five branches of the framework are covered over the five years (or complete duration) of the programme. All objectives must be developed in each year of the programme, at the appropriate level. In planning the mathematics curriculum, teachers will need to deconstruct the objectives so that they build during years 1–4 towards the highest level in the final year of the programme, providing for continuity and progression in each objective. The objectives in this guide, and the examples of interim objectives for mathematics available on the OCC, will guide teachers in making decisions about the choice of content and learning experiences offered to students, including the types of assessment that are appropriate for the students' particular stages of development.

In developing the curriculum for the different years of the programme, teachers are encouraged to plan increasingly complex tasks or units of work that will cover the entire scope of the objectives themselves. However, within these, discrete tasks or smaller units of work might concentrate on specific objectives.

In the final year of the programme, the curriculum should provide students with the **opportunity** to achieve the highest descriptor levels in the final assessment criteria (see "Mathematics assessment criteria").

The document *MYP: From principles into practice* (August 2008) provides detailed information on organizing the written, assessed and taught curriculum, including the use of interim objectives, modified assessment criteria for years 1–4 of the programme, and the planning of units of work.

Developing the curriculum within the subject

While having to meet national requirements and local standards, teachers should ensure that the curriculum they develop reflects the principles and practice of the MYP. The fundamental concepts and the IB learner profile should act as guiding principles when developing the curriculum in the school.

Teaching and learning strategies

In order to give all students opportunities to meet the MYP mathematics objectives, teachers should provide classroom environments that enhance learning and use a range of teaching and learning strategies to challenge all students.

To achieve this, MYP teachers should adopt the following strategies.

Use the areas of interaction as starting points for teaching and learning
Teaching mathematics through the areas of interaction enhances the learning experience in mathematics. The use of the areas of interaction introduces a new dimension to the inquiry and allows for a richer and in-depth exploration of concepts and topics. The areas of interaction can be used as starting points to develop units of work in mathematics, or as bridges to explore connections with other disciplines and real-world issues.

Allow students to communicate their mathematical thinking
Reading and interpreting mathematics texts, problems, functions and equations does not come naturally to most students. Some words and symbols have different meanings in mathematics and in everyday use. Many students also access the curriculum in a language other than their mother tongue. Students need to become familiar with the language of mathematics in order to communicate their ideas and findings with increasing confidence.

Teachers can help students understand the language of mathematics and master the skills of communication by providing them with tasks that allow them to read mathematics texts, to express their lines of reasoning and to communicate their findings using the appropriate mathematical language (terminology, notation, symbols) and format.

Teachers can assist students' comprehension by rephrasing instructions, speaking problems aloud and explaining their reasoning so that students learn and carry out mathematical tasks with understanding.

Devise investigations to explore mathematical concepts and ideas
MYP mathematics expects teachers to devise investigations where students choose their own strategies and methods while attempting to solve problems. Investigations can involve real-life situations or purely mathematical ones. MYP mathematics emphasizes open-ended investigations where more than one answer is possible.

Use real-life contexts and situations
When students solve problems that have been framed in real-life contexts or that are relevant to their interests, they make connections between what they learn in the classroom and its applications to other subjects and the real world. Connecting mathematical ideas and concepts to other subjects and real-life contexts enhances the understanding that learning mathematics is meaningful and functional. This allows students to reason and use mathematics when solving problems in mathematics and in other contexts.

In general, good practice in mathematics teaching is changing. Some teaching practices that have become more effective for increasing students' understanding of mathematics are listed in the following table. These changes should be reflected in the MYP classroom.

How is mathematics teaching changing?	
Increased emphasis on	**Decreased emphasis on:**
connecting mathematical concepts and applications	treating mathematics as isolated concepts and facts
developing mathematical understanding through the development of reasoning and analytical skills, making mathematics more meaningful to students	rote practice, memorization and symbol manipulation
solving real-life problems in which the context is relevant to the student	word problems as problem-solving

How is mathematics teaching changing?	
Increased emphasis on	**Decreased emphasis on:**
instruction that builds on what students know and need to learn	instruction focused on what students do not know
a variety of strategies for possible multiple solutions	one method, one answer
students being encouraged to speculate and pursue ideas	the teacher as the sole authority for providing the right answers
explaining processes in a clear and logical way and reflecting upon results	finding answers
teachers working in teams with colleagues from their own and other subject groups	teachers working in isolation
multiple sources and resources for learning	a textbook-driven curriculum
students investigating, questioning, discussing and justifying or proving	the use of exercise sheets
practical activities, including groups or collaborative tasks according to the activity	a "chalk and talk" lesson format
assessment as an integral part of instruction (formative assessment)	final examinations
a broad range of assessment strategies, including tests where students have to show their reasoning.	short-answer, multiple-choice assessment.

Developing units of work

When planning a unit of work in mathematics, teachers should ensure that:

- relevant aspects of the unit of work are presented through the perspective of at least one of the areas of interaction
- mathematical knowledge, understanding and skills are being developed
- interdisciplinary teaching is explored and used where appropriate
- differentiated instruction and diverse teaching strategies are used to cater for inquiry-based learning and multiple levels of ability
- real-life situations are used as the context for mathematics tasks, where appropriate
- local and/or global issues are used to promote inquiry into the role of mathematics in society and the environment
- tasks allow students to think about the problem-solving processes, reflect upon their methods and results, and explore the connection with everyday life
- assessment tools such as assessment rubrics, with clear descriptions of assessment outcomes, are shared with all students and these outcomes reflect the MYP mathematics aims and objectives (see "Aims and objectives")

- learner outcomes match the MYP objectives (see objectives in “Aims and objectives”) and are considered throughout the five years of the programme
- student achievement of the objectives is measured against the assessment criteria (see “Mathematics assessment criteria”).

Addressing the areas of interaction

The areas of interaction provide contexts through which teachers and students consider teaching and learning, approach the disciplines, and establish connections across disciplines. They are organizing elements that strengthen and extend student awareness and understanding through meaningful exploration of real-life issues. All teachers share the responsibility of using the areas of interaction as a focus for their units of work.

The process of inquiring into the subject content through the different perspectives or contexts of the areas of interaction enables students to develop a deeper understanding of the subject as well as the dimensions of the areas of interaction. Through this inquiry cycle of understanding and awareness, reflection and action, students engage in reflection and metacognition, which can lead them from academic knowledge to thoughtful action, helping to develop positive attitudes and a sense of personal and social responsibility.

The document *MYP: From principles into practice* (August 2008), in the section “The areas of interaction”, provides further information relating to the dimensions of each area of interaction, the inquiry cycle, planning units of work, and focusing relevant content through these areas of interaction.

There are five areas of interaction.

- Approaches to learning (ATL)
- Community and service
- Health and social education
- Environments
- Human ingenuity

The following sections on the areas of interaction provide sample questions that might be used to develop **MYP unit questions** or as **inquiry cycle questions**, depending on the content being taught. These particular questions are “content free”, and when devising their own questions, teachers can relate them to the specific content that is being explored in a unit of work.

It is important to note that the areas of interaction are ways of looking at content: some of the examples that follow could easily fit into more than one area of interaction perspective, and also have the potential to be explored through subjects other than mathematics.

The contexts that frame the content curriculum in mathematics must be natural and meaningful. Often when designing a unit of work, the context for the content will emerge naturally. To provide meaningful learning experiences, teachers should ensure that the MYP unit question gives students scope for inquiry into the issues and themes within the content. The area of interaction will then give direction to teacher-directed and student-initiated inquiry.

Please note that any reference to “I” in the areas of interaction questions could also be interpreted as “we” where this is more appropriate to the social ethos of the school or location.

Approaches to learning

How do I learn best?
How do I know?
How do I communicate my understanding?

Approaches to learning (ATL) are central to all MYP subject groups and the personal project. Through ATL, schools provide students with the tools to enable them to take responsibility for their own learning. This involves articulating, organizing and teaching the skills, attitudes and practices that students require to become successful learners.

The MYP has identified seven groups of skills that encompass ATL: organization, collaboration, communication, information literacy, reflection, thinking and transfer. The school community will need to spend time defining the ATL attitudes, skills and practices that it considers important within these groups, both for an individual subject group and across subject groups.

ATL skills area	Examples of student learning expectations	Sample questions
Organization	Time management Self management	How can I plan and organize my learning more effectively?
Collaboration	Work in groups	What are effective ways of working with my classmates? How can collaborative work improve my mathematics skills?
Communication	Mathematical literacy—know, interpret and use mathematics-specific language and forms of representation Communicating ideas clearly and logically	How is communication in mathematics different from that in other subjects? How can I ensure others understand what I mean?
Information literacy	Collecting, selecting and organizing information from a variety of sources using a range of technologies	How can ICT help my mathematics learning?
Reflection	Evaluating results and processes Evaluating my own learning	What is the value of reflection in mathematics? How can I learn in mathematics? How do I learn best in mathematics?
Thinking	Understanding and applying knowledge and concepts Identifying and selecting strategies to solve problems	How can learning mathematics improve my thinking skills?
Transfer	Using mathematical skills and knowledge in real-life contexts and making connections with other areas of knowledge	What skills are specific to mathematics? How is learning in mathematics similar or different from learning in other subjects? How does learning mathematics help me with learning in other subjects? What skills and knowledge can I take from other subjects and use in mathematics?

Some ideas that could be used to develop ATL skills through mathematics include:

- using deductive reasoning to solve a contextual problem that analyses and interprets information presented in tables, charts and graphs from various resources such as newspapers, magazines and other publications
- using open-ended investigations that have more than one possible solution and allow for more than one possible problem-solving strategy to encourage divergent thinking
- using Escher tessellations to examine geometry and design principles, and exploring how mathematics can be used to create artistic designs and effects
- using games of chance to gain insight into probabilities and the chances of an event occurring
- using real-life problems such as traffic jams, queues in the supermarket or games situations to design mathematical models based on probabilities and plan solutions to these problems
- using networks and flow diagrams as tools for making decisions for planning a travel itinerary
- using the concept of algorithm for planning and scheduling tasks for the personal project.

Community and service

How do we live in relation to each other?
How can I contribute to the community?
How can I help others?

The emphasis of community and service is on developing community awareness and a sense of belonging and responsibility towards the community so that students become engaged with, and feel empowered to act in response to, the needs of others.

Community and service starts in the classroom and extends beyond it, requiring students to discover the social reality of self, others and communities. This, in turn, may initiate involvement and service in the communities in which they live. Reflection on the needs of others and the development of students' ability to participate in and respond to these needs both contribute to the development of caring and responsible learners.

Students will explore the nature of past and present communities through mathematics, as well as their place in their own communities. Incorporating community and service into the study of mathematics encourages responsible citizenship as students deepen their knowledge and understanding of the world around them.

Examples of student learning expectations		Sample questions
Awareness and understanding of:	the concept of community—including what "community" means, how communities are different and how they are similar, what makes a community individuals in communities—including the role of the individual, the needs of the individual, the responsibilities of communities to their members different communities—including the various forms of community, the needs of different communities, the issues within the communities, organizations within communities	How is the knowledge of mathematics useful in communities? How can a community influence the learning of mathematics?

Reflection on:	attitudes—including reflection upon different social patterns and ways of life, showing initiative responsibilities—including the ethical implications of activity or inactivity within the community, using personal strengths to enhance communities, identifying personal strengths and limitations	What is the role of mathematics in a community? What would the world be like without mathematics?
Involvement through service in terms of:	community involvement—including types of involvement, effects on communities at various levels, personal involvement being an active contributor—including showing willingness and the skills to respond to the needs of others, coming up with solutions to actively resolve issues within communities.	How can I contribute to my community through mathematics? How can I improve my community through what I learn in mathematics?

Ideas that may be considered to integrate community and service through mathematics include:

- organizing a fundraising event in the school to raise money for a charity; preparing a simple budget, estimating expenses, incomes and profit for the various activities
- using tests to measure the fitness of different groups of a community; analysing results, considering age, activity, smoking habits, and so on; communicating the results using comparative tables and graphs, and developing posters to raise awareness of the importance of fitness for a healthy society
- using local newspapers to analyse articles related to statistics and social issues, and discussing how statistics can inform as well as mislead
- using a local road safety leaflet to explore concepts of speed, acceleration, distance and displacement; producing leaflets for the community to raise awareness of the importance of reducing speed around school areas
- investigating the financial effect of illegal downloading on music publishers, film companies and software publishers.

Health and social education

How do I think and act?
How am I changing?
How can I look after myself and others?

This area of interaction is about how humanity is affected by a range of social issues (including health). It includes an appreciation of these effects in various cultural settings and at different times. It is concerned with physical, social and emotional health and intelligence—key aspects of development leading to a complete and balanced lifestyle.

<table>
<tr><th colspan="2">Examples of student learning expectations</th><th>Sample questions</th></tr>
<tr><td>Awareness and understanding of:</td><td rowspan="2">ourselves in the wider society—including issues such as freedom, government health policies and globalization

ourselves and others—including issues such as relationships, sex and death

ourselves—including issues such as personal management, self-esteem and growing up

looking after ourselves—including issues such as personal hygiene, diseases and substance abuse</td><td>How does mathematics impact on society? On individuals? On me?

Can mathematics be used to influence the health of a society?

To what extent can mathematics contribute to the well-being of people and societies?

How can mathematics help to communicate the health of a society and/or nation?</td></tr>
<tr><td>Reflection on:</td><td>In what ways does mathematics allow me to express myself?

How does mathematics enable me to learn about myself and others?</td></tr>
<tr><td>Making choices in terms of:</td><td>ourselves in the wider society—including behavior and ethics

ourselves and others—including personal values and taking responsibility

understanding ourselves—including self-control or needs and wants

looking after ourselves—including diet and exercise</td><td>How can my learning in mathematics help me to make healthy choices?</td></tr>
</table>

Ideas that may be considered to integrate health and social education into mathematics include:

- investigating proportions and ratios—looking at food dishes from different cultures and performing calculations for scaling up recipes for the whole class
- using observations—investigating traffic through observations, as well as data analysis and statistics, to promote a road safety campaign around the school
- investigating population growth—using data analysis, statistics and probability to compare growth rates of different countries
- investigating the process of encrypting and decrypting data, or the flow of traffic through one-way streets, using discrete mathematics
- using mathematical functions to predict the spread of a disease or the behaviour of a population
- discussing the role of statistics and probability for providing information, its power and reliability.

Environments

What are our environments?
What resources do we have or need?
What are my responsibilities?

This area of interaction considers environments to mean the totality of conditions surrounding us—natural, built and virtual. It focuses on the wider place of human beings in the world and how we create and affect our environments. It encourages students to question, to develop positive and responsible attitudes, and to gain the motivation, skills and commitment to contribute to their environments.

Examples of student learning expectations		Sample questions
Awareness and understanding of:	the roles our environments play in the lives and well-being of humankind the effects of one environment on another the effects of our actions, attitudes and constructs, such as sustainable development and conservation physical, social, political, economic and cultural dimensions the nature and role of local and international organizations responsible for protecting our natural environments how organizational policies in one environmental dimension can affect other environments	In what way can mathematics influence natural, built and virtual environments? How does mathematics influence the school environment? What issues do natural, built and virtual environments present for mathematics? How can mathematics affect our understanding of different environments?
Reflection on:	our responsibilities to our environments the role of virtual environments in modelling our other environments	How do my mathematics skills enable me to understand different environments?
Taking action on:	a range of issues related to environments	How can my mathematics skills help me to improve my environments? What power can mathematics give us to communicate environmental issues to the world?

Ideas that may be considered to promote environmental awareness, responsibility, action and reflection in mathematics include:

- investigating natural resources—using techniques for measuring and analysing data to formulate questions and make predictions about the use and availability of a given resource at a given time in the future
- developing practical projects—using geometry and trigonometry to respond to the specific needs of local environments (town planning, designing and making models of real or imagined buildings, or other space-management applications)

- estimating water use—using mathematics to estimate water consumption and put forward plans for its management and conservation both at school and at home
- investigating household or school waste—predicting the amount of community waste to devise a campaign to raise awareness of waste management by suggesting ways of reducing, reusing or recycling waste and optimizing packaging design to reduce the loss of natural resources
- investigating endangered species—using statistics and probability to estimate the population size of a given species at a given time, to raise awareness for the conservation and protection of species in their natural environments
- investigating climate change—using data analysis and probability to discuss the evidence of the correlation between the emission of greenhouse gases and the rise in global temperature
- using modelling to predict or interpret natural patterns—for example, coastal erosion and rising sea levels due to global warming
- investigating the use of social networking through statistical analysis.

Human ingenuity

Why and how do we create?
What are the consequences?

Human ingenuity looks at human contributions in the world both in their particular context and as part of a continuing process. It stresses the way humans can initiate change, whether for good or bad, and examines the consequences (intended and unintended). This area also emphasizes both the importance of researching the developments made by people across place, time and cultures, and the importance of taking time to reflect on these developments.

Examples of student learning expectations		Sample questions
Awareness and understanding of:	the meaning of “ingenious” a range of systems, solutions and products the processes involved in innovation, creation, development and change the individual desire to create, develop or change things how systems or products develop and change over time	What is mathematics? Where does it come from? How has mathematics evolved over time? How can mathematics initiate change?

Examples of student learning expectations		Sample questions
Reflection on:	the impact of innovation and creation on individuals, communities, societies and the world the products of innovation, creation and development in context how subjects have “ways of thinking” a range of systems, solutions and products	What would the world be like without mathematics? In what ways have humans shaped mathematics? In what ways has mathematics shaped our lives? How does mathematics affect language? In what ways has knowledge influenced mathematics? In what ways has mathematics influenced knowledge? How is mathematics developing in my time and culture? How useful is mathematics in my life? What contributions has mathematics made to human civilizations?
Taking action to:	create solutions and products to solve my own and others’ problems think creatively.	

Mathematics is one of the greatest intellectual achievements of humankind and, as such, it provides many opportunities to incorporate human ingenuity into the curriculum. Some possible ideas include:

- investigating the history and evolution of mathematics across cultures and its impact on individuals and societies
- researching mathematical inventions—for example, the universality of mathematics as a language, the invention of the concept of zero
- exploring the application of mathematical knowledge in industrial and technological developments
- developing mathematical models and formulae to describe real-life phenomena
- developing mathematical ideas, such as symmetry in geometry, through the creation of tessellations
- exploring mathematical projects with interdisciplinary themes—for example, investigating the links between geometry and art/architecture, or shapes and motifs of different cultures.

Mathematics framework

The framework for MYP mathematics outlines five branches of mathematical study.

- **Number**
- **Algebra**
- **Geometry and trigonometry**
- **Statistics and probability**
- **Discrete mathematics**

Schools can use the framework for mathematics as a tool for curriculum mapping when designing and planning their mathematics courses. Schools are not expected to address all the branches of the framework in each year of the programme, nor are they asked to cover every concept and skill suggested in the framework. However, it is important that, over the five years (or complete duration) of the programme, students experience learning in all five branches of the framework for mathematics.

In this section, concepts and skills for each of the five branches of the framework for mathematics are suggested. The concepts and skills are **examples** of what students may expect to study at the two levels: standard mathematics and extended mathematics.

Concepts and skills

Number

The ability to work with numbers is an essential skill in mathematics. Students are expected to have an understanding of number concepts and to develop the skills of calculation and estimation. Students should understand that the use of numbers to express patterns and to describe real-life situations goes back to humankind's earliest beginnings, and that mathematics has multicultural roots.

Concepts	Skills
Standard and extended mathematics	
Forms of numbers: integers, fractions, decimals, exponents, standard form (scientific notation) and surds/radicals Number systems: set of positive integers and zero ($\mathbb{N}$), integers ($\mathbb{Z}$), rationals ($\mathbb{Q}$), irrationals ($\mathbb{Q}'$) and real numbers ($\mathbb{R}$)	Ordering numbers Transformation between different forms of numbers Simplification of numerical expressions in the number systems and forms of number Recognizing and classifying numbers in different number systems
The four number operations	Using the four number operations (addition, subtraction, multiplication and division) with integers, decimals and simple fractions

Concepts	Skills
Prime numbers and factors, including greatest common divisor and least common multiple	Representing a number as the product of its prime factors and using this representation to find the greatest common divisor and least common multiple
Number lines	Expressing the solution set of a linear inequality on the number line
Estimation	Using different forms of rounding, decimal approximation and significant figures Using appropriate forms of rounding to estimate results
Units of measurement	Converting between different units of measurement and between different currencies
Ratio, percentage, direct and inverse proportion	Dividing a quantity in a given ratio Finding a constant of proportionality, setting up equations and graphing direct and inverse relationships
Number sequences	Predicting the next term in the number sequence (linear, quadratic, triangular, Fibonacci)
Extended mathematics only	
Fractional exponents	Using the rules of indices to simplify numerical expressions involving radicals and exponents
Absolute and percentage error in estimations	Calculating the absolute and percentage error

Algebra

Algebra is an abstraction of the concepts first used when dealing with number and is essential for further learning in mathematics. Algebra uses letters and symbols to represent number, quantity and operations, and employs variables to solve mathematical problems.

Students who wish to continue studying mathematics beyond the MYP will require knowledge of concepts and skills in algebra. Teachers should, where appropriate, assist students' understanding of algebra by using real-life contexts for the application of algebraic knowledge and skills in problem-solving situations.

Concepts	Skills
Standard and extended mathematics	
Addition, subtraction, multiplication and division of algebraic terms	Expanding and simplifying algebraic expressions
Factorization of linear and quadratic expressions	Factorizing algebraic expressions
Substitution	Using substitution to evaluate expressions
Rearranging algebraic expressions	Changing the subject of the formula

Concepts	Skills
Algebraic fractions	Solving equations involving algebraic fractions
Integer exponents (including negative number exponents)	Using the laws of exponents
Patterns and sequences	Finding and justifying or proving general rules/ formulae for sequences
Functions: • types of functions—linear, quadratic • domain and range	Solving the linear function $f(x) = mx + c$, its graph, gradient and y-intercept Graphing different types of functions and understanding their characteristics Determining the range, given the domain
Graphs	Sketching and interpreting graphs
Equations: • linear • simultaneous • quadratic	Solving equations algebraically and using graphs
Inequalities	Solving and graphing linear inequalities
Extended mathematics only	
Logarithms and exponents: • fractional exponents • logarithms with different base numbers (including natural logarithms)	Using the laws of logarithms
Functions and graphs: • types of functions: trigonometric, exponential, logarithmic, reciprocal function $f(x) = \frac{1}{x}$ and their transformations, the square root function • inverse and composite function	Graphing different types of functions and understanding their characteristics Determining inverse and composite functions and their graphs
Equations involving the functions above	Solving equations algebraically and using graphs
Inequalities	Solving non-linear inequalities Linear programming
Arithmetic and geometric series	Developing and justifying or proving general rules/formulae for sequences Finding the sum of the series Finding unknowns (ratio, term, and so on)
Matrices	Performing basic operations with matrices Using matrices (solving equations, transformations, growth models, and so on)

Geometry and trigonometry

The study of geometry and trigonometry enhances students' spatial awareness and provides them with the tools for analysing, measuring and transforming geometric quantities in two and three dimensions.

Concepts	Skills
Standard and extended mathematics	
Geometrical elements and their classification	Naming and classifying different geometrical elements (point, line, angle, regular and irregular planar figures, solids)
Distance	Measuring distance (between two points, between a line and a point)
Angle properties	Solving problems using the properties of: • angles in different figures or positions • acute, right and obtuse angles in triangles • angles in intersecting and parallel lines • angles in regular and irregular polygons • angles in circles
Triangle properties	Solving problems involving triangles by using: • Pythagoras' theorem and its converse • properties of similar triangles • properties of congruent triangles
Perimeter/area/volume	Finding the perimeter (circumference), area and volume of regular and irregular two-dimensional (2D) and three-dimensional (3D) shapes
The Cartesian plane	Identifying the different components of the Cartesian plane: axes, origin, coordinates (x, y) and points Understanding and using the Cartesian plane, plot graphs and measuring distances between points
Trigonometric ratios in right-angled triangles	Relating angles and sides of right-angled triangles using sines, cosines and tangents Solving problems in right-angled triangles using trigonometric ratios
Constructions	Using geometry tools to make basic constructions and using these in solving problems
Simple isometric transformation	Transforming a figure by rotation, reflection, translation and enlarging
Loci	Using the concept of locus to solve problems in two dimensions

Concepts	Skills
Extended mathematics only	
Vectors and vector spaces	Adding, subtracting and scalar multiplication of vectors
Similarity and congruence theorems	Justifying or proving theorems for congruence, similarity, shape and angles
Trigonometric ratios for angles bigger than 90°	Justifying or proving simple trigonometric identities to simplify and solve equations where $0° \leq \theta \leq 360°$
Sine and cosine rules	Using the sine and cosine rules to solve problems

Statistics and probability

This branch of mathematics is concerned with the collection, analysis and interpretation of quantitative data and uses the theory of probability to estimate parameters, discover empirical laws, test hypotheses and predict the occurrence of events.

Through the study of statistics, students should develop skills associated with the collection, organization and analysis of data, enabling them to present information clearly and to discover patterns. Students will also develop critical-thinking skills, enabling them to differentiate between what happens in theory (probability) and what is observed (statistics).

Students should understand both the power and limitations of statistics, becoming aware of their legitimate use in supporting and questioning hypotheses, but also recognizing how statistics can be used to mislead as well as to counter opinions and propaganda.

Students should use these skills in their investigations and are encouraged to use information and communication technology (ICT) whenever appropriate.

Concepts	Skills
Standard and extended mathematics	
Graphical analysis and representation (pie charts, histograms, line graphs)	Constructing and interpreting graphs
Population sampling	Selecting samples and making inferences about populations
Measures of central tendency/location (mean, mode, median, quartile, percentile)	Calculating the mean, median and mode, and choosing the best measure of central tendency
Measures of dispersion (range, interquartile range)	Calculating the extent of the interquartile range
Probability of an event Probability of exclusive and combined events Probability of successive trials	Calculating probabilities of simple events Calculating probabilities of mutually exclusive events and combined events Using tree diagrams to determine the probability of repeated events

Concepts	Skills
Extended mathematics only	
Normal distribution and standard deviation	Making inferences about normal distributed data given the mean and the standard deviation
Linear regression Correlation	Drawing the line of best fit
Conditional probability	Calculating conditional probability

Discrete mathematics

Discrete mathematics is a relatively new branch of mathematics that has its roots in abstract algebra and has adopted the language and notation of graph theory. Discrete mathematics contributes to the understanding of systems and formal structures. Its techniques have become increasingly important for people to analyse and solve problems in technology, science, engineering, business and other complex systems.

Students should develop logical-thinking skills and be able to articulate their understanding through the use of Venn diagrams, structure diagrams and flow charts. Discrete mathematics provides new approaches to learning (ATL) in the MYP.

Students should be aware of the real-world applications of discrete mathematics, which may include road or rail networks, computer networks, communications networks, optimal routes, time- and project-management techniques, and critical path analysis.

Concepts	Skills
Standard and extended mathematics	
Sets Venn diagrams	Performing operations Drawing and interpreting Venn diagrams Using Venn diagrams to solve problems in real-life contexts
Logic	Expressing ideas in two-value systems (Boolean algebra) Applying truth tables to determine the truth for complex statements
Networks (including trees)	Locating paths and tours Analysing networks to find complete paths, shortest distance paths Solving problems involving optimal solutions Devising and describing procedures for performing complete calculations Using networks and flow charts to solve problems in real-life contexts
Algorithms	Analysing and using well-defined procedures for solving complex problems

Concepts	Skills
Extended mathematics only	
Topology	Classifying and describing topological objects and simplifying knots
Directed networks	Performing critical path analyses
Codes and ciphers	Encoding and decoding information using translations and modular arithmetic Generating and translating bar codes Encoding and decoding with RSA codes that involve public and private keys

Assessment in the MYP

There is no external assessment provided by the IB for the MYP and therefore no formal externally set or marked examinations. All assessment in the MYP is carried out by teachers in participating schools and relies on their professional expertise in making qualitative judgments, as they do every day in the classroom. In line with the general IB assessment philosophy, a norm-referenced approach to assessment is not appropriate to the MYP. Instead, MYP schools must follow a criterion-related approach. This means that students' work must be assessed against defined assessment criteria and not against the work of other students.

The IB moderation and monitoring of assessment procedures ensure that the final judgments made by these teachers all conform to an agreed scale of measurement on common criteria.

It is expected that the procedures for assessment and the MYP assessment criteria are shared with both students and parents as an aid to the learning process.

Using the assessment criteria

The assessment criteria published in this guide correspond to the objectives of this subject group. The achievement levels described have been written with year 5 final assessment in mind.

All schools **must** use the assessment criteria published in this guide for final assessment, although local or national requirements may involve other assessment models and criteria as well.

In years 1–4, schools should modify the descriptors of the achievement levels for each criterion according to the progression of learning organized by them and guided by the interim objectives. These modified criteria must be based on the MYP principles of assessment and must provide for a coherent approach to assessment practices over the entire programme. Schools may add other criteria, in addition to the MYP criteria, in response to national requirements and report on these internally to parents and students.

Clarifying published criteria in year 5

During the final year of the programme, the final assessment criteria as published in each subject-group guide must be used when awarding levels. However, specific expectations of students for a given task must still be defined.

Teachers will need to clarify the expectations of any given task with direct reference to the published assessment criteria. For example, in mathematics, teachers would need to clarify exactly what "appropriate deductions" means in the context of a given assessment task. This might be in the form of:

- a task-specific clarification of the criteria, using the published criteria but with some wording changed to match the task
- an oral discussion of the expectations
- a task sheet that explains the expectations.

It is important that teachers specify the expected outcomes at the beginning of each individual task so that students are aware of what is required.

When clarifying expectations for students, teachers must ensure that they do not alter the standard expected in the published criteria, nor introduce new aspects. When awarding levels in year 5, teachers themselves should always use the published criteria.

Please also see the "Mathematics: Moderation" section for guidance on what is required as part of background information.

The "best-fit" approach

The descriptors for each criterion are hierarchical. When assessing a student's work, teachers should read the descriptors (starting with level 0) until they reach a descriptor that describes an achievement level that the work being assessed has **not** attained. The work is therefore best described by the preceding descriptor.

Where it is not clearly evident which level descriptor should apply, teachers must use their judgment to select the descriptor that best matches the student's work overall. The "best-fit" approach allows teachers to select the achievement level that best describes the piece of work being assessed.

If the work is a strong example of achievement in a band, the teacher should give it the higher achievement level in the band. If the work is a weak example of achievement in that band, the teacher should give it the lower achievement level in the band.

Further guidance

Only whole numbers should be recorded; partial levels, fractions and decimals are not acceptable.

The levels attributed to the descriptors must not be considered as fixed percentages, nor should it be assumed that there are arithmetical relationships between descriptors. For example, a level 4 performance is not necessarily twice as good as a level 2 performance.

Teachers should not think in terms of a pass or fail boundary for each criterion, or make comparisons with, or conversions to, the IB 1–7 grade scale, but should concentrate on identifying the appropriate descriptor for each assessment criterion.

The highest descriptors do not imply faultless performance, but should be achievable by students at the end of the programme. Teachers should therefore not hesitate to use the highest and lowest levels if they are appropriate descriptors for the work being assessed.

A student who attains a high achievement level for one criterion will not necessarily reach high achievement levels for the other criteria. Similarly, a student who attains a low achievement level for one criterion will not necessarily attain low achievement levels for the other criteria.

Teachers should not assume that the results of a group of students being assessed will follow any particular distribution plan.

Further information on MYP assessment can be found in the document *MYP: From principles into practice* (August 2008) in the section "Assessment".

Mathematics assessment criteria

Please note that the assessment criteria in this guide are for first use in **final assessment** in 2012.

The following assessment criteria have been established by the IB for mathematics in the MYP. All final assessment in the final year of the MYP must be based on these assessment criteria even if schools are not registering students for IB-validated grades and certification.

Criterion A	Knowledge and understanding	Maximum 8
Criterion B	Investigating patterns	Maximum 8
Criterion C	Communication in mathematics	Maximum 6
Criterion D	Reflection in mathematics	Maximum 6

For each assessment criterion, a number of band descriptors are defined. These describe a range of achievement levels with the lowest represented as 0. The criteria are not equally weighted.

The descriptors concentrate on positive achievement, although failure to achieve may be included in the description for the lower levels.

Criterion A: Knowledge and understanding

Maximum: 8

Knowledge and understanding are fundamental to studying mathematics and form the base from which to explore concepts and develop skills. This criterion expects students to use their knowledge and to demonstrate their understanding of the concepts and skills of the prescribed framework in order to make deductions and solve problems.

This criterion examines to what extent the student is able to:

- know and demonstrate understanding of the concepts from the five branches of mathematics (number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics)
- use appropriate mathematical concepts and skills to solve problems in both familiar and unfamiliar situations, including those in real-life contexts
- select and apply general rules correctly to make deductions and solve problems, including those in real-life contexts.

Assessment tasks for this criterion are likely to be classroom tests, examinations, real-life problems and investigations that may have a variety of solutions.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student **generally** makes appropriate deductions when solving **simple** problems in **familiar** contexts.
3–4	The student **generally** makes appropriate deductions when solving **more complex** problems in **familiar** contexts.
5–6	The student **generally** makes appropriate deductions when solving **challenging** problems in a **variety** of **familiar** contexts.
7–8	The student **consistently** makes appropriate deductions when solving **challenging** problems in a **variety** of contexts including **unfamiliar situations**.

Notes

- **Unfamiliar situation**: challenging questions or instructions set in a new context in which students are required to apply knowledge and/or skills they have been taught.
- **Deduction**: reasoning from the general to the particular/specific to reach a conclusion from the information given.
- **Context**: the situation and the parameters given to a problem.

Criterion B: Investigating patterns

Maximum: 8

Through the use of mathematical investigations, students are given the opportunity to apply mathematical knowledge and problem-solving techniques to investigate a problem, generate and/or analyse information, find relationships and patterns, describe these mathematically as general rules, and justify or prove them.

This criterion examines to what extent the student is able to:

- select and apply appropriate inquiry and mathematical problem-solving techniques
- recognize patterns
- describe patterns as relationships or general rules
- draw conclusions consistent with findings
- justify or prove mathematical relationships and general rules.

Assessment tasks for this criterion should be mathematical investigations of some complexity, as appropriate to the level of MYP mathematics. Tasks should allow students to choose their own mathematical techniques to investigate problems, and to reason from the specific to the general. Assessment tasks could have a variety of solutions and may be set in real-life contexts. Teachers should clearly state whether the student has to provide a justification or a proof.

Teachers should include a good balance between tasks done under test conditions and tasks done at home in order to ensure the development of independent mathematical thinking.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student **applies**, **with some guidance**, mathematical problem-solving techniques to recognize simple patterns.
3–4	The student **applies** mathematical problem-solving techniques to recognize patterns, **and suggests** relationships or general rules.
5–6	The student **selects and applies** mathematical problem-solving techniques to recognize patterns, **describes** them as relationships or general rules, and **draws conclusions** consistent with findings.
7–8	The student **selects and applies** mathematical problem-solving techniques to recognize patterns, **describes** them as relationships or general rules, draws the **correct conclusions** consistent with the correct findings, and **provides justifications or a proof**.

Notes

- **Pattern**: the underlining order, regularity or predictability between the elements of a mathematical system. To identify pattern is to begin to understand how mathematics applies to the world in which we live. The repetitive features of patterns can be identified and described as relationships or generalized rules.
- **Justification**: give valid reasons or evidence to support the conclusion and explain **why** the rule works.

- **Proof**: a mathematical demonstration of the truth of the relationship or general rule.
- A student who describes a general rule consistent with incorrect findings will still be able to achieve in the 5–6 band, provided that the rule is of an equivalent level of complexity.
- Clear guidance should be incorporated in the teachers' task to ensure all students receive the same guidance and understand the basic requirements of the task.

Criterion C: Communication in mathematics

Maximum: 6

Students are expected to use mathematical language appropriately when communicating mathematical ideas, reasoning and findings—both orally and in writing.

This criterion examines to what extent the student is able to:

- use appropriate mathematical language in both oral and written explanations
- use different forms of mathematical representation
- communicate a complete and coherent mathematical line of reasoning using different forms of representation when investigating problems.

Students are encouraged to choose and use ICT tools as appropriate and, where available, to enhance communication of their mathematical ideas. Some of the possible ICT tools used in mathematics include spreadsheets, graph plotter software, dynamic geometry software, computer algebra systems, mathematics content-specific software, graphic display calculators (GDC), word processing, desktop publishing, graphic organizers and screenshots.

Assessment tasks for this criterion are likely to be real-life problems, tests, examinations and investigations.

Tests and examinations that are to be assessed against criterion C must be designed to allow students to show complete lines of reasoning using mathematical language.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student shows **basic use** of mathematical language and/or forms of mathematical representation. The lines of reasoning are **difficult to follow**.
3–4	The student shows **sufficient use** of mathematical language and forms of mathematical representation. The lines of reasoning are **clear though not always logical or complete**. The student moves between different forms of representation with **some success**.
5–6	The student shows **good use** of mathematical language and forms of mathematical representation. The lines of reasoning are **concise, logical and complete**. The student moves **effectively** between different forms of representation.

Notes

- **Mathematical language**: the use of notation, symbols, terminology and verbal explanations.
- **Forms of mathematical representation**: refers to formulae, diagrams, tables, charts, graphs and models used to represent mathematical information.

Criterion D: Reflection in mathematics

Maximum: 6

MYP mathematics encourages students to reflect upon their findings and problem-solving processes.

This criterion examines to what extent the student is able to:

- explain whether his or her results make sense in the context of the problem
- explain the importance of his or her findings in connection to real life where appropriate
- justify the degree of accuracy of his or her results where appropriate
- suggest improvements to the method when necessary.

Assessment tasks are most likely to be mathematical investigations or real-life problems. Generally these types of tasks will provide students with opportunities to use mathematical concepts and skills to solve problems in real-life contexts.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors below.
1–2	The student **attempts to explain** whether his or her results make sense in the context of the problem. The student **attempts to describe** the importance of his or her findings in connection to real life where appropriate.
3–4	The student **correctly but briefly explains** whether his or her results make sense in the context of the problem. The student **describes the importance** of his or her findings in connection to real life where appropriate. The student **attempts to justify** the degree of accuracy of his or her results where appropriate.
5–6	The student **critically explains** whether his or her results make sense in the context of the problem. The student provides a **detailed explanation** of the importance of his or her findings in connection to real life where appropriate. The student **justifies** the degree of accuracy of his or her results where appropriate. The student suggests improvements to his or her method where appropriate.

Notes

- **Explain**: give a detailed account including reasons or causes.
- **Describe**: give a detailed account.

Determining the final grade

This section explains the process by which a student's overall achievement level (in terms of the assessment criteria) is converted to a single grade.

1. Collecting the information

Teachers will use assessment tasks to make judgments of their students' performance against the assessment criteria at intervals during the final year in the subject. Many of the assessment tasks will allow judgments of levels to be made with regard to more than one criterion.

For the purposes of final assessment, teachers **must** ensure that, for each student, they make **several judgments against each criterion**. This can be achieved by using some kinds of assessment task more than once, or by incorporating other types of assessment activity. MYP mathematics has **four** criteria and so **at least eight** judgments (two per criterion) must be made for each student in the final year for the purposes of final assessment. However, as more complex tasks will allow students to be assessed against several criteria, final assessment may rest on a limited number of tasks.

Important: If more than one teacher is involved in one subject for a single year group, the school must ensure **internal standardization** is used to provide a common system for the application of the assessment criteria to each student. In joint assessment, internal standardization is best achieved by:

- the use of common assessment tasks
- shared assessment between the teachers
- regular contact between the teachers.

In certain schools, students may be grouped according to ability within the same subject. In such cases, the teachers' final assessment of student performance across all groups must be based on a **consistent application of the assessment criteria to all students**. A different standard should not be applied to different groups.

2. Making a final judgment for each criterion

When the judgments on the various tasks have been made, teachers will be in a position to establish a final profile of achievement for each student by determining the **single most appropriate level for each criterion**. Where the judgments for a criterion differ for specific assessment tasks, the teacher must decide which level best represents the student's final standard of achievement.

Important: Teachers should not average the levels gained in year 5 for any given criterion. Students can develop academically right up to the end of the programme, and teachers must make a professional judgment (that is also supported by work completed) as to which level best corresponds to a student's general level of performance for each of the criteria towards the end of the programme.

3. Determining the final criterion levels total

The final levels for each criterion must then be added together to give a **final criterion levels total** for mathematics for each student. In mathematics, students have the opportunity to gain a maximum level of 8 for criteria A and B and 6 for criteria C and D. Therefore the maximum final criterion levels total for mathematics will be 28.

The final criterion levels total is the total that will be submitted to the IB via IBIS (IB information system) for those schools that have registered students to receive IB-validated grades.

4. Determining the final grade for mathematics

Grade boundaries must be applied to the criterion levels totals to decide the final grade for each student.

Please see the MYP *Coordinator's handbook* for the table of grade boundaries for mathematics.

All MYP subjects receive final grades in the range from 1 (lowest) to 7 (highest) on the IB record of achievement, where students have been registered for IB-validated grades. The general MYP grade descriptors describe the achievement required for the award of the subject grade. After using the conversion table to determine a student's final mathematics grade, teachers should check the general grade descriptor table to ensure that the description equally reflects the student's achievement.

> Schools requiring **IB-validated grades** are required to use **only** the published MYP subject-specific criteria as a basis for the final results that they submit to the IB (both for moderation and as final assessment for certification).
>
> Other schools (those not requiring IB-validated grades) will use the published criteria together with any additional criteria that they have developed independently, and report internally to students and parents. These schools may decide on their own grade boundaries (if using published and additional criteria), or use the boundaries published by the IB.

Mathematics: Moderation

The following details apply **only** to schools that request **IB-validated grades**.

Please ensure that you also refer to the section "Assessment in the MYP".

Purpose of moderation

The external moderation procedure in all MYP subjects and the personal project exists to ensure that students from different schools and different countries receive comparable grades for comparable work, and that the same standards apply from year to year.

All MYP assessment is carried out by the students' own teachers (or by the supervisors in the case of the personal project). The IB moderation procedures ensure that the final tasks set by those teachers are appropriate and that the final judgments made by these teachers all conform to an agreed scale of measurement on common criteria.

To ensure this comparability and conformity, moderation samples submitted to the IB **must** be assessed using the assessment criteria and achievement levels listed in this guide.

The submission date for moderation samples is before the end of a school's academic year. Tasks submitted for moderation are not absolutely final tasks. Schools must continue to make further assessments of students' work after moderation samples have been submitted, as these later tasks will also contribute towards the student's final criterion levels total.

For general information on moderation, please see *MYP: From principles into practice* (August 2008), section "Moderation".

Teachers should note that there are three distinct phases to the moderation process.

- Phase 1: Submission of moderation samples
- Phase 2: Submission of criterion levels totals
- Phase 3: Award of MYP grades

Phase 1: Submission of moderation samples

Schools that request IB-validated grades for their students must register these students following the guidelines in the MYP *Coordinator's handbook*. This includes students who are only eligible for the record of achievement along with those who are also eligible for the MYP certificate.

Each moderation sample must include **eight folders of students' work** with each folder representing the work of a single student. The selection of student work should be representative of a range of abilities within the final year group, comprising two comparatively good folders, four folders showing average ability and two comparatively weak folders. Only the work of students registered for IB-validated grades should be submitted. If there are fewer than eight students registered, the sample will therefore have fewer than eight folders.

Since June 2006, schools that have had minimal adjustments to their results over a three-year period have been instructed to send only four folders of student work instead of eight in the relevant subjects. "Minimal adjustments" means differences between teachers' and moderators' totals of within plus or minus 3. This does not mean that there will be no changes to final grades, as some students' totals will still cross grade boundaries even though the differences, and therefore the moderation factors applied, are small. Schools are advised via the moderation reports whether they can send four folders the following year. The situation is monitored annually and applies only to the subjects that have been identified in the moderation reports. For further information, please contact your MYP coordinator.

Prescribed minimum tasks

There must be **two** judgments **only** for each mathematics criterion (A, B, C, D) entered on the moderation coversheet contained in each student folder. The following pieces must be submitted in each folder.

- A **broad-based classroom test/examination** composed of a range of questions and problems, in familiar and unfamiliar situations, covering at least **three** of the branches of the framework for mathematics, and which allows students to reach all levels of achievement. (*Criterion A is strongly recommended as one of the criteria used to assess this task.*)
- A **mathematical investigation**, done under test conditions, where students are given the opportunity to recognize patterns, describe them as relationships or general rules and justify or prove them. (*Criterion B is strongly recommended as one of the criteria used to assess this task.)*
- A **real-life problem** where students are given the opportunity to apply mathematics to a real-life context, reflect upon and evaluate their findings. (*Criterion D is **essential** as one of the criteria used to assess this task.*)

For the moderation of extended mathematics, teachers must indicate in the assessment tasks which topics of the extended part of the framework are being assessed.

Should **a fourth task** be necessary to meet the requirements of two judgments against each criterion, one of the following tasks may be included in the moderation sample.

- A second test/examination that does not have to be broad-based
- A second mathematical investigation that does not have to be done under test conditions
- A second real-life problem

For moderation purposes, it is suggested that schools submit a maximum of four tasks.

Characteristics of the prescribed tasks for moderation

Broad-based classroom test/examination

This task should allow students to demonstrate knowledge and understanding of at least three branches of the framework for mathematics and should consist of questions and problems set in both familiar and unfamiliar situations. Papers consisting exclusively of multiple-choice questions and/or questions requiring "true" or "false" answers are not appropriate, as they do not provide students with the opportunity to demonstrate their understanding through the use of mathematical lines of reasoning. Criterion A should be one of the criteria used to assess this task.

Mathematical investigation

This task should develop from an initial problem that does not have an obvious solution or approach.

The investigation:

- should provide a challenge and an opportunity for creativity
- should allow students to choose different courses of action from a range of options
- may have a variety of answers
- should develop the skills of:
  - producing a strategy
  - generating data
  - recognizing patterns or structures
  - searching for further cases
  - formulating, testing and justifying or proving a general rule.

Mathematical investigations should be designed and carried out independently by the student. Teacher-guided tasks where students simply follow a procedure are not suitable tasks.

Criterion B should be one of the criteria used to assess this task.

Real-life problem

This task could be inspired by the relationship between mathematics and other areas of knowledge such as the sciences, the physical world, the environment, the economy, technology, health, medicine or society. The task should also highlight the role of mathematics in the real world.

This task should allow the student to apply mathematics to real life by:

- identifying the problem
- translating the problem into mathematics
- solving the problem
- interpreting the solutions in the real-life context.

It is **essential** to include criterion D as one of the criteria used to assess this task.

Notes

- The work in the moderation sample should be taken from the same unit(s) of work for all students, as far as possible.
- Student work submitted for moderation should reflect the types of tasks used for final assessment and must be devised to give students the opportunity to reach the highest descriptors of each criterion.

- In order to help schools with the timing of the preparation of moderation samples, work from the end of year 4 of the programme can be included, provided the final-year assessment criteria have been used. The sample must also include work produced in year 5.
- Although group work is encouraged in practice, group work must not be submitted for moderation purposes. It is difficult for moderators to ascertain a student's actual contribution to a piece of work that was undertaken in a group situation.
- In law, students retain copyright in work they create themselves, and the school probably retains copyright in the tasks created by teachers. However, when the school submits this work to the IB, students and schools are deemed to be granting the IB a non-exclusive worldwide licence to use the work. Please see the MYP *Coordinator's handbook*, sections F1 and F3, for further information on how this work may be used, and section F4 for the *Student claim of exclusive copyright* form if needed.
- Each criterion (A, B, C, D) must have **two** judgments made against it for the purposes of moderation.
- Teachers may have devised tasks to focus on only one or two of the criteria, and therefore do not make two judgments against each criterion with the three prescribed minimum tasks. In this case it is acceptable to enclose additional tasks in the sample until each criterion has been assessed twice. If a criterion has been assessed more than twice in the sample, the extra assessment(s) will not be moderated.

Practical organization of the moderation sample

- The coversheet *Form F3.1* must be used to record the judgments for each criterion in each student's folder.
- Background information should be compiled in an additional folder to the students' folders. It should document details that will be useful to the moderators.
  - The context and expected outcomes of the unit of work
  - Time allocation
  - The degree of teacher support
  - The conditions under which the work was completed
  - Information about the application of the assessment criteria

  Unit planners must be included in the background information in order to give moderators an idea of the context in which the task was set. The moderator will not make comments about the unit planner.
- Background information in mathematics should also indicate which concepts and skills were the specific focus of the assessment tasks.
- Background information should be compiled into a ninth folder. This information does not need to be added into each of the eight student folders. The background information must be submitted in the working language of the school (English, French or Spanish).
- Clear and legible copies of work should be submitted in the sample. Original work may be submitted but it is not returned to schools.
- Students are expected to reference sources they use for their work as a matter of course.
- If teachers and students use third-party material as stimuli and/or as part of their tasks, this material must be fully referenced. This will include the title of the source, the author, the publication date, the publisher and, for books only, the ISBN. Examples of third-party material include newspaper and magazine articles, cartoons, videos, movie excerpts, extracts from books, pictures (please check the acknowledgments in the original publication for the original sources), diagrams, graphs, tables, statistics, materials from websites, and so on.

Phase 2: Submission of criterion levels totals

Phase 1 of the moderation process takes place before the end of most schools' academic year. After submitting moderation samples, teachers should continue to assess students' work until **final assessment**.

After final assessment, teachers should use the procedure described in "Determining the final grade" to arrive at a **criterion levels total** for each student registered for certification.

The MYP coordinator will then enter each registered student's criterion levels total on **IBIS**, and submit this to the IB.

Phase 3: Award of MYP grades

Following moderation in each subject, the IB may, where appropriate, apply a moderation factor to the criterion levels totals submitted by a school. Final grades will then be determined by applying grade boundaries to these moderated totals.

Schools will receive notification of the final grades for their students and the IB will also provide a general and a school-specific moderation report for each subject in which students were registered.

The MYP *Coordinator's handbook* provides further guidelines on submitting criterion levels totals in each subject.

Mathematics: Monitoring of assessment

The following details apply to schools **not** requesting IB-validated grades.

Please ensure that you also refer to the sections "Assessment in the MYP" and "Mathematics: Moderation".

Definition

Monitoring of assessment is a service available to IB World Schools offering the MYP, whereby schools can send samples of assessed student work to the IB to receive feedback from an experienced MYP moderator in the form of a report. This service is subject to a fee.

Monitoring of assessment is aimed at providing support and guidance in the implementation and development of the programme with regard to internal assessment procedures and practices. It is not linked to validation of students' grades, and therefore differs from the process of external moderation. Monitoring of assessment is currently limited to assessment conducted in the final three years of the programme.

Samples for monitoring of assessment in mathematics must be submitted in English, French or Spanish, although these may be translations into one of these languages.

Details on registering for monitoring of assessment and fees, as well as the latest updated versions of the coversheets, are available in the MYP *Coordinator's handbook*.

Further information on monitoring of assessment can be found in the document *MYP: From principles into practice* (August 2008), in the section "Monitoring of assessment". Brief information follows here.

Purpose

There are three reasons why schools send in a monitoring of assessment sample.

- As a requirement for the school's programme evaluation visit
- As a pre-check before sending in samples for moderation
- To receive guidance on a particular subject

Choice of tasks for monitoring of assessment

For evaluation visit and general advice

Schools can decide on the types of task they wish to submit for monitoring of assessment for the evaluation visit or for general advice. However, they are recommended to consider the prescribed minimum tasks detailed in the "Mathematics: Moderation" section, as this is designed to give an even spread over the mathematics assessment criteria.

Prior to moderation

If the school is requesting monitoring of assessment in preparation for future moderation, the tasks in the following list **must** be included in the sample of assessed student work. These are the required minimum tasks listed in the "Mathematics: Moderation" section.

- A **broad-based classroom test/examination** composed of a range of questions and problems, in familiar and unfamiliar situations, covering at least **three** of the branches of the framework for mathematics, and which allows students to reach all levels of achievement. (*Criterion A is strongly recommended as one of the criteria used to assess this task.*)
- A **mathematical investigation**, done under test conditions, where students are given the opportunity to recognize patterns, describe them as relationships or general rules and justify or prove them. (*Criterion B is strongly recommended as one of the criteria used to assess this task.*)
- A **real-life problem** where students are given the opportunity to apply mathematics to a real-life context, reflect upon and evaluate their findings. (*Criterion D is* ***essential*** *as one of the criteria used to assess this task.*)

Please see the "Mathematics: Moderation" section for further notes and information.

MYP mathematics frequently asked questions

General

How is MYP mathematics different from other mathematics courses?

MYP mathematics aims to provide students with the mathematical competency and the intellectual capacity for pursuing further studies in mathematics and for becoming confident users of mathematics later in life. Therefore, MYP mathematics should be accessible to and be studied by all students.

Unlike other mathematics courses, where the emphasis is placed solely on developing mathematical knowledge and skills, MYP mathematics focuses on the development of an intellectual capacity for lifelong learning. The final objectives for MYP mathematics support the IB learner profile by promoting the development of students who are knowledgeable, inquirers, communicators and reflective thinkers.

MYP mathematics encourages the use of investigations as a means to support inquiry-based learning, while allowing students to explore concepts and solve problems using mathematical skills and reasoning. Communication and reflection are fundamental to the teaching and learning of MYP mathematics, and contribute to the development of analytical and critical thinkers who can access information and communicate ideas and findings confidently using the language of mathematics.

MYP mathematics encourages teachers to use problems and investigations set in real-life contexts, to help students see the connections between mathematics and other areas of life, and to appreciate the role of mathematics in life and society.

How does the MYP prepare students for the Diploma Programme?

MYP mathematics aims to prepare students for further post-16 courses in mathematics, including the mathematics standard level (SL) and mathematics higher level (HL) courses of the Diploma Programme (DP).

The MYP framework for mathematics was devised to provide sufficient breadth and depth to meet the needs of students wishing to study mathematics at DP level.

The framework for mathematics was revised in conjunction with the DP curriculum to cover the concepts and skills of the presumed knowledge (PK) for courses at mathematics SL and HL. The two levels of the MYP framework (standard and extended) were refined to allow a smooth transition from MYP mathematics to DP mathematics courses.

Can I teach to objectives other than those listed in the *Mathematics guide*?

Teachers may teach to objectives in addition to those listed. However, students must be given the opportunity to achieve all of the objectives listed in this guide by the end of the final year of the MYP.

What level of mathematics should we offer? Do we need to separate students by ability?

The MYP framework for mathematics allows students to work at two levels: standard and extended mathematics. However, it is the prerogative of the school to decide whether to offer one or both levels, and whether to group students by ability or to teach mixed-ability classes.

These decisions should be based on the analysis of a number of factors, including local and national requirements, the characteristics of the student population, and the availability of resources in the school.

The majority of schools offering both standard mathematics and extended mathematics do not teach students of different ability in different classes until they are in MYP year 3.

Students aspiring to study mathematics HL as part of the DP would be best advised to study MYP extended mathematics. However, many schools prepare students for DP mathematics HL within the MYP standard mathematics course.

Should schools address every branch of the framework in every year of the programme?

No, it is not a requirement for schools to address all the branches of the framework in each year of the programme. Nor are schools expected to cover every concept and skill of the five branches in their school syllabus, as if they were ticking boxes.

Schools should decide how to structure and sequence their instruction, using the framework as a guideline, so that the courses they design best address the needs of their students while also complying with local and national requirements. For example, schools may want to place a stronger emphasis on some branches in the early years and leave the others for later exploration, or they may prefer to address some concepts and skills across all branches as students progress through the MYP.

However, in order to provide students with opportunities to meet the final objectives of MYP mathematics, schools must ensure that, over the five years (or complete duration) of the programme, students experience learning in number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics.

How much of the framework for mathematics needs to be covered at the different levels?

The framework for mathematics should not be confused with a syllabus where teachers are expected to cover all the topics in a course. The framework should act as a guide to help teachers design and plan their courses at both levels. The framework for mathematics is structured into five branches and lists suggested concepts and skills for each branch for both standard and extended levels.

The mathematics curriculum that the school designs must allow students to experience learning in all five branches of the framework by the end of the programme. Schools must also ensure that the courses developed do not compromise the final objectives of MYP mathematics.

How can I help students who have difficulties with the level of standard mathematics?

Some students may need additional support to meet the objectives of MYP standard mathematics. In this case it is important to determine students' needs in order to best support their learning. Students may be accessing the course in a language other than their mother tongue, or may have a diagnosed or non-diagnosed special educational need (SEN). Any of these cases needs to be investigated and identified so that schools can organize appropriate support systems for these students. This may include teacher training, language support courses, subject support tutorials, peer mentoring, differentiated instruction, modification of assessment tasks, or an interview with an SEN specialist to make a diagnosis and guide further instruction.

For further information on these issues, please refer to the document *Learning in a language other than mother tongue in IB programmes* and to the SEN page, resources and forums on the online curriculum centre (OCC).

How can I adapt mathematics teaching to meet the needs of students with special educational needs?

There are a number of ways in which teachers can differentiate their instruction in order to meet students' special educational needs in mathematics. Teachers should focus on the development of basic skills and competencies to support students' mathematical ability as part of lifelong learning skills for the workplace.

Some suggestions for differentiated instruction in mathematics are listed below.

- Use real-life contexts for the application of knowledge and skills when devising problem-solving situations, in order to provide meaningful experiences for students.
- Differentiate tasks to incorporate the wide range of students' skills and capabilities.
- Use direct instruction when teaching basic skills, in order to help students process the vocabulary, text format, structure and use of symbols in mathematics.
- Encourage cooperative learning strategies and an individualized approach.
- Encourage repetition and allow additional time for consolidation and reinforcement of knowledge.
- Limit the number of exercises and concentrate on enhancing understanding. Do not expect students with SEN to do as many problems as others; limit the number required to show understanding and application.
- Use ICT (for example, calculators, mathematical software) to aid and support understanding.
- Develop a tiered approach to knowledge to suit different students' needs.
    - Core (basic) knowledge that has to be known by all
    - Standard knowledge that should be known by most
    - More challenging and "interest-based" knowledge for those who wish to stretch themselves
- Promote the design of projects that offer choices related to the students' interests (for example, analysing costs of various mobile phone providers).
- Monitor and record progress on an ongoing basis and devise yearly plans targeting different groups of students.

For more strategies please refer to the SEN page, resources and forums on the OCC.

How can I detect plagiarism? How can I avoid it in the first place?

If you suspect that work has been plagiarized, one way to check is to conduct an internet search. Using a major search engine, type in a selection of the work in inverted commas (one sentence should be sufficient). If the work has been taken directly from a website it will be detected. Your school may also subscribe to a plagiarism detection site. Plagiarism from other sources can be more difficult to detect, depending on how familiar the teacher is with all the resources available to the students.

The best solution is to avoid setting tasks that are easy to complete through plagiarism, or other forms of academic dishonesty. Tasks should be challenging, but not so difficult that students are tempted to use dishonest means to complete them, and support should be available when students require it. For further information on IB policy on academic honesty, please see the publication *Academic honesty* available on the online curriculum centre (OCC).

Does the IB recommend any particular style of referencing/quoting/footnoting?

There is no set style for referencing in the MYP. Schools need to decide on one or more recognized styles of referencing that suits the needs of the students and the school.

Can we use teaching resources if we do not have a clear idea of where the resources came from?

Teachers need to adhere to the guidelines of academic honesty as much as the students. Therefore teachers need to make every effort to reference and acknowledge the work of others that they use in the classroom.

Why are the overall grade boundaries not included in the guide?

The grade boundaries are included in the MYP *Coordinator's handbook* (available on the OCC), which is updated every year. This gives the flexibility to adjust grade boundaries if necessary after the first moderation session.

Assessment

I want to assess my students in a wide variety of ways without being restricted to the choice of a "test/examination", a "mathematical investigation" or a "real-life problem". Can I assess in other ways as well?

Yes. The "test/examination", "mathematical investigation" and "real-life problem" are the required minimum tasks for moderation/monitoring of assessment, and are only a snapshot of what is assessed in schools. Student ability should be assessed through a wide range of assessment activities during all years of MYP instruction.

Why change the investigation to "test conditions"?

The reasons for this decision were twofold. Firstly, to reduce the overall workload for students, and secondly, to ensure that authentic and individual work is submitted for moderation. Setting the investigation under test conditions provides time constraints and might limit students' scope for inquiry. Therefore, teachers should ensure that the tasks they design provide sufficient information for students to investigate and allow them enough time to complete their investigation during the time set for the test.

What is an open-ended investigation in mathematics?

An open-ended investigation is one that has more than one possible solution, or more than one method of approach. An example of the former could be: "Design a container that will hold 330 ml of liquid and be made from as little material as possible." An example of the latter could be: "From the past 100 years of records in the 1,500 metre run, predict what the record will be in 2050."

What is the difference between "apply" and "select and apply" in the context of the wording of criterion B?

In levels 3–4, students apply a mathematical problem-solving technique with some guidance. In this context "apply" implies that the student may have received some help in the form of instructions for the problem or from the teacher.

However, in order to achieve levels 5–6, students are expected to "select and apply" mathematical problem-solving techniques without guidance. "Select" in this context implies that the student has considered different alternatives and has consciously and independently decided which mathematical technique to use.

What is the connection between the criterion levels and the final grade?

A criterion level only gives a partial assessment of mathematics. For example, a level for criterion C only shows a student's achievement in "communication in mathematics", and does not give an overall picture of his or her mathematical competence. To work out a student's final grade, a teacher must take into account levels from all of the criteria, giving a balanced final result. In summary, the final grade is an overall view of the student's achievement in the subject; the criterion levels show a student's achievement in components of the subject.

For example:

	Criterion A (/8)	Criterion B (/8)	Criterion C (/6)	Criterion D (/6)	Levels total (/28)	Final grade
Student 1	5	4	6	6	21	5
Student 2	8	8	3	3	22	5

Criterion levels and final grades are useful in different ways. For example, schools may use final grades for reporting to parents, but use criterion levels in designing their lessons, as these give more specific feedback on the needs of the students.

When is "where appropriate" appropriate?

The teacher must identify when an assessment strand is appropriate to the task. It is essential to allow students to address all strands over each year of study. Please note that it is essential to assess the following strand of criterion D in all real-life problems:

*"The student provides a **detailed explanation** of the importance of their findings in connection to real life where appropriate."*

Moderation

What are the requirements for moderation of extended mathematics?

Extended mathematics has the same moderation requirements as standard mathematics. Both have to meet the prescribed minimum assessment tasks as listed in the "Mathematics: Moderation" section of this guide. However, for those tasks submitted for **extended mathematics**, teachers must clearly indicate in the tasks which topics of the extended framework have been assessed.

How can I address each criterion twice with the three required tasks for moderation?

When designing a task, teachers should check the criteria descriptors to ensure that the task is suitable for assessment against the desired criteria and that it allows students the opportunity to reach the highest achievement levels.

Although, in theory, any type of task could be made appropriate for any of the four criteria, a broad-based test would be most appropriate for assessment against criterion A, a mathematical investigation for the assessment against criterion B, and a real-life problem for the assessment against criterion D, as shown in the table below.

Task	A	B	C	D
Broad-based test	✓			
Mathematical investigation		✓		
Real-life problem				✓

In order to provide two judgments per criterion using the minimum number of assessment tasks, teachers should explore how to adapt the tasks so that they can be assessed against more than one criterion.

A test can generally be written in such a way that it can also be assessed against communication (criterion C), but it would be difficult to assess a test against criteria B and D reliably. A mathematical investigation could be designed to be assessed against criteria B, C and D, while a real-life problem could be written so that it is also suitable for assessment against criteria A and C.

Task	A	B	C	D
Broad-based test	✓		(✓)	
Mathematical investigation		✓	(✓)	(✓)
Real-life problem	(✓)		(✓)	✓

✓ suggested
(✓) could possibly be assessed

If two judgments against each criterion cannot be made sensibly using three assessment tasks, a fourth one may be submitted.

What is "background information"? What should I include?

Background information is the information provided in a moderation or monitoring of assessment sample that tells the moderator or assessor details of the tasks, what the expectations were, what resources were available and under what conditions the tasks were completed. Examples of background information include worksheets, instructions or notes given to students, information on time allocation/length of preparation, degree of teacher or peer support allowed, blank copies of tasks, and comments on student work. In mathematics it is important that background information indicates the degree of assistance the students received when working on the tasks for moderation. Responses to suggestions made in the previous moderation report, or monitoring of assessment, must be included whenever relevant.

If teachers use third-party material either as stimuli, as part of their tasks or both, this material must be fully referenced. This will include the title of the source, the author, the publication date, the publisher and, for books only, the ISBN. Examples of third-party material include newspaper and magazine articles, cartoons, videos, movie excerpts, extracts from books, pictures (please check the acknowledgments in the publication for the original sources), diagrams, graphs, tables, statistics, materials from websites, and so on.

If the sample differs from the stated requirements in any way, this should also be explained in the background information.

If the tasks I give students are not appropriate, will my students be penalized?

If the tasks submitted for moderation do not give the students the opportunity to demonstrate all of the skills listed in the corresponding criterion/criteria, or if the standard of performance expected is too low for final-year students, then the task will be deemed inappropriate.

When tasks are inappropriate, it is often the case that the levels awarded by the teacher are too high. In these cases, the levels will be lowered appropriately, and this may result in students' final grades also being lowered.

Do all the tasks that we send for moderation have to involve ICT and third-party media/resources?

No, this is not a moderation requirement. ICT should be used as and when appropriate to support learning, enhance understanding or make communication more effective. Naturally, if third-party resources are used in tasks, students and teachers are expected to acknowledge the sources according to a recognized convention.

MYP mathematics glossary

Context	The situation and the parameters given to a problem.
Deductive reasoning	Reasoning from the more general to the specific.
Direct instruction (SEN)	Teaching approach where the teacher will transmit information directly to the students, generally in a very structured way (step-by-step approach). The teacher provides immediate feedback, modelling situations and allowing time for independent practice. Direct instruction is particularly effective for teaching basic skills.
Forms of (mathematical) representation	Refers to formulae, diagrams, tables, charts, graphs and models used to represent mathematical information.
General rules	These include formulae, theorems, proven relationships and laws.
Inductive reasoning	Involves making generalizations from specific observations, recognizing patterns and making general rules.
Justification	A clear and logical mathematical explanation of **why** a rule works.
Mathematical investigation	An investigation that requires students to reason from the specific to the general; may have a variety of solutions and may be set in a real-life situation.
Mathematical language	The use of notation, symbols, terminology and verbal explanations.
Mixed-ability classroom	Students with all abilities share the same classroom; no grouping or teaching differentiation is applied.
Pattern	The underlying order, regularity or predictability between the elements of a mathematical system.
Proof	A mathematical demonstration of the truth of a given proposition.
Unfamiliar situation	This refers to challenging questions or instructions set in a new context in which students are required to apply knowledge and/or skills they have been taught.

MYP mathematics example interim objectives

Objectives for years 1, 3 and 5 of the Middle Years Programme

Year 5 objectives

The mathematics objectives for year 5 can be found in the section on "Mathematics in the MYP" earlier in this guide. This set of **prescribed** objectives forms the basis for the **assessment criteria**, also published in the guide, which must be used for final assessment of students' work during year 5.

Example interim objectives

Example interim objectives for years 1 and 3 of the MYP appear in the tables that follow. They have been developed in order to:

- promote articulation between the MYP and the Primary Years Programme (PYP)
- support individual schools in developing a coherent curriculum across the five years of the programme (or however many years a school is authorized to offer)
- emphasize the need to introduce students to the required knowledge, understanding, skills and attitudes from the first year of the programme
- provide examples of possible learning experiences that will allow students to work towards meeting the final objectives for year 5
- support schools that are authorized to offer the first three years of the MYP in designing appropriate assessment tasks for the end of the third year.

Unlike the objectives for year 5, the interim objectives for years 1 and 3 are not prescribed**,** although the IB recommends that all schools use them. Schools may choose to adopt the objectives contained in this document or develop their own.

If choosing to develop their own interim objectives, schools must start with the prescribed objectives for year 5 and modify each one by taking into account the age, prior knowledge and stage of development of students in an earlier year of the programme. Each year 5 objective will then correspond directly to a modified objective in a preceding year of the programme. **No objectives should be omitted** from a previous year as it is vital to ensure a coherent progression of learning across all five years of the programme.

MYP units of work

Examples of possible learning experiences, each aligned to an objective, appear in the tables that follow. Each learning experience is intended to form part of a larger unit of work that is designed to address a central question or theme, known as the **MYP unit question**. More information about MYP units of work can be found in the section on "Planning for teaching and learning" in *MYP: From principles into practice* (August 2008).

Within each unit of work, the **context for learning**, **significant concept(s)** and **assessment tasks** are defined in relation to the MYP unit question. The areas of interaction provide the context for learning while the significant concepts refer to the underlying concepts that define the principal goal of the unit. Assessment tasks are designed to address the levels of students' engagement with the MYP unit question and the aligned objectives.

Context for learning

Every MYP unit of work has an approaches to learning (ATL) component: a shared and agreed set of skills that all teachers develop with their students throughout the entire programme. The context that frames a particular unit of work is generally derived from one of the other four areas of interaction, although ATL might be the specific context on some occasions. Some of the examples of learning experiences listed in the tables that follow have an obvious connection to one of the areas of interaction—for example, investigating the relationship between the volume of air in a classroom (and other enclosed spaces) and the health requirements of each student. Other connections may become clear only after a more considered approach, but teachers should be able to establish these connections for their own students within each MYP unit of work.

Several examples of learning experiences listed also strongly suggest the possibility of planning an interdisciplinary unit in collaboration with other subject teachers, for example, representing Newton's laws of motion as algebraic equations, tables and graphs using data that has been generated and data that has been collected experimentally.

Assessment tasks

One of the first stages in planning a unit of work is to design **summative assessment tasks**, linked to the MYP unit question, which provide varied opportunities for students to demonstrate their knowledge, understanding, skills and attitudes. It is also important to include ongoing **formative assessment tasks** within a unit of work as these provide valuable insights into the extent of student learning as the unit of work progresses.

It is important to realize that the formats of both summative and formative assessment tasks need not be reduced to examinations, tests, quizzes and written questions set as homework. These formats are valid in certain cases but do not always take into account different learning styles and may not provide students with sufficient creative scope to demonstrate all they have learned. There are many different ways in which evidence of student learning can be found. For example, students could carry out assessment tasks that involve:

- making a presentation using visual aids (for example, flipcharts, electronic slides)
- solving a cross-number puzzle where the clues are provided in the form of calculations to be made and/or problems to be solved
- playing a game that requires a particular set of skills or knowledge and understanding of certain concepts
- making a three-dimensional model (for example, scale models of the earth, moon and sun)
- telling a story (for example, stories where numbers have been deliberately scaled up or down by multiples of ten to provide comic entertainment, thereby demonstrating the need for accuracy with regard to place value)
- keeping a personal journal that documents their development of mathematical understanding
- making a poster or wall chart
- writing a short song or poem that incorporates important mathematical principles (for example, a rap chant incorporating the principles of Pythagoras' theorem)

- creating a mnemonic as an aid to memory (for example, the rules for the sine, cosine and tangent properties of a right-angled triangle expressed in one word, SOHCAHTOA, or as a phrase or saying, "**S**ome **o**wls **h**ave ....")
- keeping a scrapbook containing extracts from the media that illustrate a particular mathematical property
- maintaining a folder of their own work
- developing an information booklet/leaflet that describes a concept and/or mathematical process in detail
- writing a summary sheet as a revision guide for a particular mathematical topic
- creating pictures, diagrams or cartoons to illustrate a particular concept or process
- carrying out an investigation
- collecting data and storing it in appropriate formats (for example, tables, spreadsheets)
- creating a personal data booklet.

Tables of objectives

A Knowledge and understanding

Knowledge and understanding are fundamental to studying mathematics and form the base from which to explore concepts and develop problem-solving skills. Through knowledge and understanding, students develop mathematical reasoning to make deductions and solve problems.

Year 1	Year 3	Year 5
Objectives		
At the end of the first year, students should be able to demonstrate basic knowledge and understanding of the following branches of mathematics: • number • algebra • geometry and trigonometry • statistics and probability • discrete mathematics by being able to:	At the end of the third year, students should be able to demonstrate some knowledge and understanding of the following five branches of mathematics: • number • algebra • geometry and trigonometry • statistics and probability • discrete mathematics by being able to:	At the end of the course, students should be able to demonstrate knowledge and understanding of the following five branches of mathematics: • number • algebra • geometry and trigonometry • statistics and probability • discrete mathematics by being able to:
• know and demonstrate understanding of some of the basic concepts of number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics	• know and demonstrate understanding of some of the concepts of number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics	• know and demonstrate understanding of the concepts from the five branches of mathematics (number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics)
• use basic concept-specific strategies to solve simple problems in both familiar and unfamiliar situations including those in real-life contexts	• use appropriate mathematical concepts and skills to solve simple problems in both familiar and unfamiliar situations including those in real-life contexts	• use appropriate mathematical concepts and skills to solve problems in both familiar and unfamiliar situations including those in real-life contexts
• apply basic rules correctly to solve simple problems including those in real-life contexts.	• select and apply basic rules correctly to solve problems including those in real-life contexts.	• select and apply general rules correctly to make deductions and solve problems, including those in real-life contexts.

<table>
<tr><th>Year 1</th><th>Year 3</th><th>Year 5</th></tr>
<tr><td colspan="3">Examples of possible learning experiences</td></tr>
<tr>
<td>Number
Students could:
find data in newspapers and classify it as discrete or continuous
classify numbers as natural, odd, even, square and/or triangular
use the Sieve of Eratosthenes to find prime numbers less than 100
count very large sets of objects (coins, cars passing, people in a large space, numbers) to emphasize the importance of organization and grouping
make a time line on the wall of the classroom emphasizing dates of important mathematicians and mathematical discoveries.</td>
<td>Number
Students could:
make a geological time line on the wall of the classroom
investigate the relationship between the volume of air in a classroom (and other enclosed spaces) and the health requirements of each student
investigate rounding numbers to a specified number of significant figures by considering the accuracy of measurements in real life, such as the length of a 100 m athletics track, the extent of possible errors and the impact these may have.</td>
<td>Number
Students could:
compare the number of kilometres per litre of fuel per passenger used by planes, trains, buses and/or cars
explore the history and significance of irrational numbers and identify some of the symbols used for particular irrational numbers, for example, π (pi), e (Euler's number), φ (the golden ratio).</td>
</tr>
<tr>
<td>Algebra
Students could:
use a pan balance to simulate the addition or subtraction of like quantities from both sides of an equation by keeping the pans balanced
use the balance model to create equations for classmates to solve
create pictures on squared paper and provide a list of coordinates for classmates to construct the picture by joining the points in order
create a booklet containing information on algebra topics (explanations, examples and exercises) for use by subsequent students.</td>
<td>Algebra
Students could:
generate a series of ordered pairs by substituting values in a linear equation and have classmates identify patterns and/or work out the formula
find a rule for the number of 1 cm squares needed to put a 1 cm-wide frame around a square picture whose side is n cm.</td>
<td>Algebra
Students could:
show that the ratios of successive terms of a Fibonacci sequence (u_n/u_{n+1}) converge to the same value regardless of the term chosen as u_1
find the best angle for throwing a basketball so that it will go in the basket from the free throw line, by modelling its trajectory graphically using their knowledge of quadratic equations
investigate exponential growth in a biological population.</td>
</tr>
</table>

Year 1	Year 3	Year 5
Geometry and trigonometry Students could: • investigate the sum of the interior angles of triangles and quadrilaterals by drawing and cutting out different shapes on paper, tearing off the angles and fitting them together to form straight lines and/or circles • investigate the tangent ratio by comparing students' heights with the lengths of their shadows • investigate reflections by using a mirror to reflect faces along different centre lines.	**Geometry and trigonometry** Students could: • draw or construct a model of an appropriate building (the school or their own house) by piecing together rectangular or triangular prisms only • estimate the volumes of irregular solid objects as the sum of more simple approximated shapes and verify their results by immersing each object in water and measuring the displaced volume • investigate the underlying patterns and constructions evident in particular designs and/or artworks—for example, traditional Moroccan designs—and then create their own tessellated designs • use Pythagoras' theorem and the trigonometric ratios as tools for measuring large-scale objects and/or distances in open spaces.	**Geometry and trigonometry** Students could: • use the unit circle as a physical tool to calculate the values of different trigonometric ratios (in order to appreciate the circular nature and symmetry of each function, and the significance of the asymptotes in the tangent function) • use the transformations of translation, reflection, rotation, enlargement and shear to describe the actions of a particular cartoon character • carry out research into the history of angle measurement and the introduction of trigonometry.
Statistics and probability Students could: • find the ranges and means of leaf lengths on two trees of the same species but located in different environments (sun and shade).	**Statistics and probability** Students could: • design statistical surveys to investigate health and social education issues, with guidance from the teacher • construct a tree diagram for a three-day weather forecast where the probability of rain on any day is estimated from past data • compare the lengths of words and/or sentences in texts aimed at different readerships.	**Statistics and probability** Students could: • collect information relating to used cars for sale (mileage, age, make, engine size, cost now, cost when new) and explore the relationships between different pairs of variables • select several countries and look for key statistics (population growth, average income and life expectancy) on the internet in order to answer questions such as, "Do people in richer countries appear to live longer?", "Is there any relation between the size of a population and its average income?"

<table>
<tr><th>Year 1</th><th>Year 3</th><th>Year 5</th></tr>
<tr>
<td>Discrete mathematics
Students could:
• draw two large intersecting circles on the floor and identify two categories of student, for example, {girls} and {students wearing something blue}, and move into one of the four defined regions according to their characteristics
• play mathematical games (for example, “bingo”) by calculating the answers to simple problems read out by the teacher to review and reinforce previous learning.</td>
<td>Discrete mathematics
Students could:
• create a minimum network for broadband cables to connect five major cities in their country
• create a Koch snowflake by starting with a large equilateral triangle, dividing each side into three equal parts, removing the middle part and replacing it with two sides of a triangle equal in length to the part that was removed, and so on.</td>
<td>Discrete mathematics
Students could:
• solve a logical puzzle, such as the following: “You have three boxes of fruit, one with apples, one with oranges and one mixed; each box is labelled but the labels are not on the correct boxes of fruit. How can you know what each box contains simply by taking one piece of fruit from one box?”
• create their own logic puzzle by using websites such as http://www.edhelper.com/logic_puzzles.htm)
• play the chaos game (refer to http://en.wikipedia.org/wiki/Chaos_game</td>
</tr>
</table>

B Investigating patterns

Investigating patterns allows students to experience the excitement and satisfaction of mathematical discovery. Mathematical inquiry encourages students to become risk-takers, inquirers and critical thinkers. The ability to inquire is invaluable in the MYP and contributes to lifelong learning.

Through the use of mathematical investigations, students are given the opportunity to apply mathematical knowledge and problem-solving techniques to investigate a problem, generate and/or analyse information, find relationships and patterns, describe these mathematically as general rules, and justify or prove them.

Year 1	Year 3	Year 5
Objectives		
At the end of the first year, when investigating problems, in both theoretical and real-life contexts, students should be able to:	At the end of the third year, when investigating problems, in both theoretical and real-life contexts, students should be able to:	At the end of the course, when investigating problems, in both theoretical and real-life contexts, students should be able to:
• apply basic inquiry and mathematical problem-solving techniques, with guidance from the teacher, by identifying variables, posing relevant questions, organizing data and using an appropriate model	• select and apply basic inquiry and mathematical problem-solving techniques to problems by asking searching questions	• select and apply appropriate inquiry and mathematical problem-solving techniques
• recognize simple patterns similar to previously seen examples	• recognize simple patterns in different situations	• recognize patterns
• describe simple patterns in words and/or diagrams	• describe simple patterns as relationships or general rules	• describe patterns as relationships or general rules
• arrive at a result or set of results and make predictions based on extending the pattern(s)	• arrive at a single result or set of results and make predictions consistent with findings	• draw conclusions consistent with findings
• describe simple mathematical relationships.	• explain simple mathematical relationships and general rules using logical arguments.	• justify or prove mathematical relationships and general rules.
Examples of possible learning experiences		
Number Students could: • predict the next three numbers in a sequence • find large numbers that can be reduced to prime factors by their classmates • investigate how many different designs can be made by shading squares in a 3 x 3 square.	**Number** Students could: • determine ways of finding the sum of the terms in an arithmetic sequence, describing their methods in general terms • investigate the meaning of negative exponents using a calculator • investigate the patterns present in the Fibonacci sequence.	**Number** Students could: • be given the diameter of a model of the earth (a globe) as 100 cm, and investigate the diameters of corresponding models of the planets, given their true diameters expressed in standard form (a x 10_n), and discuss the status of Pluto as a planet.

Year 1	Year 3	Year 5
Algebra Students could: • investigate the graphs of $y = mx + c$ using appropriate software • measure and plot the extended length of a spring against the weight of an object hung on the end • investigate problems such as the following: "How long does it take to make *n* pieces of toast on a one-sided grill that can take two pieces at a time?"	**Algebra** Students could: • investigate the graphs of $y = (x - a)(x - b)$ and their solutions for different values of *y* using a graphic display calculator.	**Algebra** Students could: • analyse and compare male and female record times in a particular sport (running, swimming) from 1900 to the present day, and predict times for the year 2050 by investigating lines or curves of best fit for the data collected.
Geometry and trigonometry Students could: • investigate the sum of the interior angles of different *n*-sided polygons ($n > 2$) • use different instruments to measure a range of objects and discuss the ease and accuracy of each technique.	**Geometry and trigonometry** Students could: • write a logical explanation for the sum of the exterior angles of a polygon equalling 360° • investigate how to measure tall structures such as telegraph poles and buildings • use geometry to predict the angle of the noontime sun at the two solstices.	**Geometry and trigonometry** Students could: • carry out a survey of the school grounds for the purpose of creating a detailed and accurately scaled map/plan • investigate how a sphere can be projected on to a plane.
Statistics and probability Students could: • investigate the probabilities for the different outcomes when tossing two coins or rolling two dice.	**Statistics and probability** Students could: • choose, with logical explanations, which measure of central tendency would be most appropriate for typical family size, height of students, amount of pocket money.	**Statistics and probability** Students could: • design a questionnaire to elicit data reflecting attitudes to issues relevant to them; circulate this questionnaire to students in their own school and one other school, possibly in another country; then collect and compare the data • create a fundraising game of chance that is profitable and will attract players.
Discrete mathematics Students could: • use Venn diagrams to analyse different aspects of after-school activities.	**Discrete mathematics** Students could: • investigate shortest paths in local networks (bus routes, metro lines) • investigate the concept of map colouring by conducting research into the four-colour problem and attempting to illustrate this rule by colouring a map of their country showing the different regions/states.	**Discrete mathematics** Students could: • investigate whether, when you start with an odd number *a*, square it to give *b*, subtract 1 to give *c*, divide by 2 to give *d*, and add 1 to give *e*, the equation $a^2 + d^2 = e^2$ will always be satisfied.

C Communication in mathematics

Mathematics provides a powerful and universal language. Students are expected to use mathematical language appropriately when communicating mathematical ideas, reasoning and findings—both orally and in writing.

Year 1	Year 3	Year 5
Objectives		
At the end of the first year, students should be able to communicate mathematical ideas, reasoning and findings by being able to:	At the end of the third year, students should be able to communicate mathematical ideas, reasoning and findings by being able to:	At the end of the course, students should be able to communicate mathematical ideas, reasoning and findings by being able to:
• use appropriate mathematical language (notation, symbols, terminology) in both oral and written communications, with guidance from the teacher	• use appropriate mathematical language (notation, symbols, terminology) in both oral and written explanations in familiar situations	• use appropriate mathematical language (notation, symbols, terminology) in both oral and written explanations
• use different forms of mathematical representation (simple formulae, diagrams, tables, charts, graphs and models), with guidance from the teacher	• use different forms of mathematical representation (simple formulae, diagrams, tables, charts, graphs and models)	• use different forms of mathematical representation (formulae, diagrams, tables, charts, graphs and models)
• state, in writing and/or verbally, the steps followed in solving simple problems.	• communicate a mathematical line of reasoning in solving simple problems using different forms of representation.	• communicate a complete and coherent mathematical line of reasoning using different forms of representation when investigating problems.
Examples of possible learning experiences		
		General Students could: • take it in turns to summarize the important elements of selected lessons by putting the information into a class file or posting it on a website in the form of a blog; the document could then be used as a revision tool by the class.

Year 1	Year 3	Year 5
Number Students could: • modify quantities and measurements in well-known events or stories to create absurdities and then act out or model the event or story, for example, speed limits being set at 50 m per hour (instead of 50 km per hour); playing on a football pitch where it is assumed that 1 m^2 = 100 cm^2 • investigate discounts in advertisements in order to determine the best deals.	**Number** Students could: • collect media clippings of the inappropriate use of mathematical symbols and terminology, and/or inappropriate representations of data (for example, misleading labelling on graphs) and describe how these could lead to incorrect interpretations.	**Number** Students could: • be given a series of real-life calculations to carry out using a calculator (for example, "A third of the people in a city with a population of 500,000 live in poverty—how many people live in poverty?") and be asked to justify the number of significant figures given in their answers.
Algebra Students could: • explain the steps involved in solving a linear equation • explain the significance of m and c when the line represented by $y = mx + c$ is graphed.	**Algebra** Students could: • use motion recording equipment to create distance/time graphs • create feasible distance/time graphs and physically model the graphs created by others by moving around the classroom.	**Algebra** Students could: • be given an open-ended problem with fixed and variable costs, such as budgeting for an event, where the solution depends on the parameters, expected outcomes and accuracy of estimates • investigate Newton's laws of motion in the form of algebraic equations, tables and graphs using generated data and data that has been collected experimentally • investigate and describe the trajectory of small objects falling from different forms of transport (bicycles, lorries/trains, hot-air balloons).
Geometry and trigonometry Students could: • make a scale drawing of a bicycle • investigate the properties of similar two-dimensional shapes (triangles, squares and circles).	**Geometry and trigonometry** Students could: • write out directions, using bearings and distances, to describe the cycle routes in their local area • carry out a survey of their school grounds/campus in order to create an accurate plan.	**Geometry and trigonometry** Students could: • design an orienteering course on the sports field where the beginning and end points coincide, and use practical trials and/or the sine and cosine rules to demonstrate that these points are coincident.

<table>
<tr><th>Year 1</th><th>Year 3</th><th>Year 5</th></tr>
<tr>
<td>Statistics and probability
Students could:
measure the lengths of their ears and represent the data in tables and graphs, and find measures of central tendency
investigate the probability of various events taking place based on available data (for example, the probability of rain on a particular day of the year).</td>
<td>Statistics and probability
Students could:
collect data from a weather station and represent it graphically as a tool for investigating trends
discuss the advantages and disadvantages of obtaining data by sampling large populations, by referring to sampling techniques used by the media (for example, opinion polls).</td>
<td>Statistics and probability
Students could:
investigate different methods of sampling large populations (random sampling, stratified sampling, systematic sampling, cluster sampling, convenience sampling) and create a poster explaining each one.</td>
</tr>
<tr>
<td>Discrete mathematics
Students could:
use Venn diagrams to classify quadrilaterals that have equal sides, parallel lines, equal angles
draw diagrams showing three different routes they could take to travel from home to a specific destination and determine the best one by considering either times or distances.</td>
<td>Discrete mathematics
Students could:
investigate the similarities and differences between the skills needed for two different occupations using Venn diagrams (for example, music teacher and rock star)
create flow charts to describe some simple mathematical processes (for example, finding the greatest common divisor of two numbers).</td>
<td>Discrete mathematics
Students could:
conduct a poll among themselves to determine their preferences for different types of music; collate the results in the form of a Venn diagram; then describe the information displayed.</td>
</tr>
<tr><td colspan="3">Students are encouraged to choose and use information and communication technology (ICT) tools as appropriate and, where available, to enhance communication of their mathematical ideas. ICT tools can include spreadsheets, graph plotter software, dynamic geometry software, computer algebra systems, mathematics content-specific software, graphic display calculators (GDC), word processing, desktop publishing, graphic organizers and screenshots.</td></tr>
</table>

D Reflection in mathematics

MYP mathematics encourages students to reflect upon their findings and problem-solving processes. Students are encouraged to share their thinking with teachers and peers and to examine different problem-solving strategies. Critical reflection in mathematics helps students gain insight into their strengths and weaknesses as learners and to appreciate the value of errors as powerful motivators to enhance learning and understanding.

Year 1	Year 3	Year 5
Objectives		
At the end of the first year, students should be able to:	At the end of the third year, students should be able to:	At the end of the course, students should be able to:
• consider the reasonableness of their results in the context of the problem	• consider the reasonableness of their results in the context of the problem and attempt to explain whether they make sense	• explain whether their results make sense in the context of the problem
• consider the importance of their findings, with guidance from the teacher	• consider the importance of their findings	• explain the importance of their findings
• distinguish between measurement and counting, and demonstrate an appreciation of the difference between degrees of error in measuring and mistakes in counting, measuring and calculating	• consider the degree of accuracy of their results, where appropriate, and estimate errors in simple measurements	• justify the degree of accuracy of the results
• consider alternatives to the method when appropriate, with guidance from the teacher.	• consider alternatives to the method when appropriate.	• suggest improvements to the method.
Examples of possible learning experiences		
Number Students could: • estimate answers before carrying out calculations • discuss the precision of different measuring instruments (rulers, callipers, protractors, theodolites).	**Number** Students could: • investigate how well the "golden ratio" applies to famous buildings or paintings.	**Number** Students could: • discuss the appropriateness of degrees of accuracy for different data/information found in the media.
Algebra Students could: • substitute answers in original problems to check results.	**Algebra** Students could: • determine the domain and/or range of functions involving physical processes, for example, the elastic limit of a rubber band as a linear function within a limited domain.	**Algebra** Students could: • discuss when substitution, graphing or elimination are the most appropriate strategies for solving different sets of simultaneous equations • investigate whether a table-tennis ball rolled off a desk will follow a parabolic path.

Year 1	Year 3	Year 5
Geometry and trigonometry Students could: • find the least and greatest amount of paint necessary to paint their bedrooms.	**Geometry and trigonometry** Students could: • design an economical shape for a 500 ml bottle of soda • use Pick's theorem to estimate the area of regions on a map • scale up the measurements of a popular doll or toy model to determine whether their measurements lead to absurdities when their proportions are scaled to a realistic level.	**Geometry and trigonometry** Students could: • make a sundial and investigate the position of the shadow at different times of the year compared to the pre-calculated positions.
Statistics and probability Students could: • draw a bar chart showing the number of hours of television watched each day by classmates • collect examples from the media of data displayed in different ways (bar graphs, pie charts, pictograms) and comment on their effectiveness.	**Statistics and probability** Students could: • analyse the data collected to investigate an environmental problem.	**Statistics and probability** Students could: • investigate, by collecting appropriate data, whether the annual harvest of a particular fish species justifies the creation of conservation laws.
Discrete mathematics Students could: • invent a problem similar to the "Bridges of Königsberg".	**Discrete mathematics** Students could: • create and model a traffic problem involving one-way and two-way streets.	**Discrete mathematics** Students could: • devise emergency exit paths to support the swift and safe evacuation of students and staff in their school.